Math 157 - Test 2
October 23, 2013

Name key

Score

Show all work to receive full credit. Supply explanations where necessary.

1. (10 points) Let $f(x) = 10\sqrt{x}$.

   (a) Find $f'(x)$.

   $$f(x) = 10x^{1/2} \Rightarrow f'(x) = 10\left(\tfrac{1}{2}\right)x^{-1/2}$$

   $$= 5x^{-1/2} = \boxed{\frac{5}{\sqrt{x}}}$$

   (b) Find an equation of the line tangent to the graph of f at the point where $x = 4$.

   $$m = f'(4) = \frac{5}{2}$$

   $$x = 4 \Rightarrow y = 10\sqrt{4} = 20$$

   $$\boxed{y - 20 = \frac{5}{2}(x-4)}$$

   (c) Find $f''(x)$.

   $$f'(x) = 5x^{-1/2} \Rightarrow f''(x) = -\frac{5}{2}x^{-3/2} = \boxed{\frac{-5}{2\sqrt[3]{x^2}}}$$

2. (4 points) Find $\frac{d^{17}y}{dx^{17}}$ if $\underbrace{y = 3x^{10} - 8x^7 - 9x^2 + 12}$.

   EACH DERIVATIVE WILL DECREASE THE DEGREE BY 1.

   17^{TH} DERIVATIVE IS $\boxed{0}$

3. (6 points) The profit, in dollars, from selling x laptop computers is given by

$$P = -0.04x^2 + 25x - 1500.$$

Find the sales level that results in a marginal profit of zero.

$$\frac{dP}{dx} = -0.08x + 25$$

$$\frac{dP}{dx} = 0 \Rightarrow 0.08x = 25$$

$$x = \frac{25}{0.08} = \boxed{312.5}$$

4. (8 points) Suppose y is implicitly defined as a function of x by the equation

$$4x^2 = 3y^2 + xy.$$

Find dy/dx at $(1, 1)$.

$$\frac{d}{dx}(4x^2) = \frac{d}{dx}(3y^2 + xy)$$

$$8x = 6y\frac{dy}{dx} + y + x\frac{dy}{dx}$$

$$8x - y = (6y + x)\frac{dy}{dx}$$

$$\frac{8x-y}{6y+x} = \frac{dy}{dx} \Rightarrow \left.\frac{dy}{dx}\right|_{(1,1)} = \frac{8-1}{6+1} = \boxed{1}$$

5. (20 points) Compute the derivative of each function.

(a) $f(x) = \dfrac{4}{x} + 9 = 4x^{-1} + 9$

$$\boxed{f'(x) = -4x^{-2}}$$

(b) $g(t) = (t^2 - 2t + 5)(t^3 - 1)$

$$\boxed{g'(t) = (2t-2)(t^3-1) + (t^2-2t+5)(3t^2)}$$

(c) $y = \dfrac{x^2 + 6x + 5}{\sqrt[5]{x}}$

$$\boxed{\frac{dy}{dx} = \frac{(x^{1/5})(2x+6) - (x^2+6x+5)\left(\frac{1}{5}x^{-4/5}\right)}{\left(\sqrt[5]{x}\right)^2}}$$

(d) $f(x) = (4x^{-1} - x^4 + 2)^5$

$$\boxed{f'(x) = 5(4x^{-1} - x^4 + 2)^4(-4x^{-2} - 4x^3)}$$

6. (8 points) An object is moving along the graph of $y^2 - 5x^2 = -1$ in such a way that $\frac{dx}{dt} = -3$. Find $\frac{dy}{dt}$ at the point $(1, 2)$.

$$\frac{d}{dt}\left(y^2 - 5x^2\right) = \frac{d}{dt}(-1)$$

$$2y\frac{dy}{dt} - 10x\frac{dx}{dt} = 0$$

WHEN $(x,y) = (1,2)$...

$$2(2)\frac{dy}{dt} - 10(1)(-3) = 0$$

$$4\frac{dy}{dt} + 30 = 0$$

$$\frac{dy}{dt} = \frac{-30}{4} = \boxed{-\frac{15}{2}}$$

7. (8 points) A national distributor of pet toys determines the cost and revenue functions for one of its toys:

$$C = 1.2x - 0.0001x^2, \quad 0 \le x \le 6000$$
$$R = 3.6x - 0.0005x^2, \quad 0 \le x \le 6000$$

Determine the open interval(s) on which the profit is increasing.

$$P = R - C = (3.6x - 0.0005x^2) - (1.2x - 0.0001x^2)$$

$$P = 2.4x - 0.0004x^2$$

$$P' = 2.4 - 0.0008x$$

$$P' = 0 \Rightarrow x = 3000$$

P' IS POS　　P' IS NEG

0　　3000　　6000

P IS INC　　P IS DEC

PROFIT IS INCREASING ON $(0, 3000)$

8. (12 points) A woman jumps off a diving board and soars through the air in such a way that her height, in feet, at time t (in seconds) is given by

$$h = -16t^2 + 16t + 32.$$

(a) Find the diver's velocity function.

$$\boxed{h'(t) = -32t + 16}$$

(b) When does the diver reach her highest point?

$$h'(t) = 0 \Rightarrow -32t + 16 = 0$$
$$\Rightarrow \boxed{t = \tfrac{1}{2} \text{ SEC}}$$

(c) What is the diver's average velocity over the first 2 seconds?

$$\frac{h(2) - h(0)}{2} = \frac{[-16(4) + 16(2) + 32] - [32]}{2}$$
$$= -16 \text{ FT/SEC}$$

(d) When does the diver hit the water?

$$h(t) = 0 \Rightarrow -16(t^2 - t - 2) = 0$$
$$-16(t-2)(t+1) = 0 \Rightarrow \boxed{t = 2 \text{ SEC}}$$

9. (8 points) An oil tanker has run aground and ruptured its hull. Leaking oil is spreading in all directions. The polluted region is a circle, which is growing steadily at a rate $35\,\text{m}^2/\text{hr}$. How fast is the radius of the oil slick growing at the moment when the radius is $50\,\text{m}$? (The area of a circle is given by $A = \pi r^2$.)

A = AREA OF POLLUTED REGION AT TIME t

r = RADIUS OF POLLUTED REGION AT TIME t

$\frac{dA}{dt} = 35$, FIND $\frac{dr}{dt}$ WHEN $r = 50$

$$A = \pi r^2$$

$$\frac{dA}{dt} = 2\pi r \frac{dr}{dt}$$

WHEN $r = 50$...

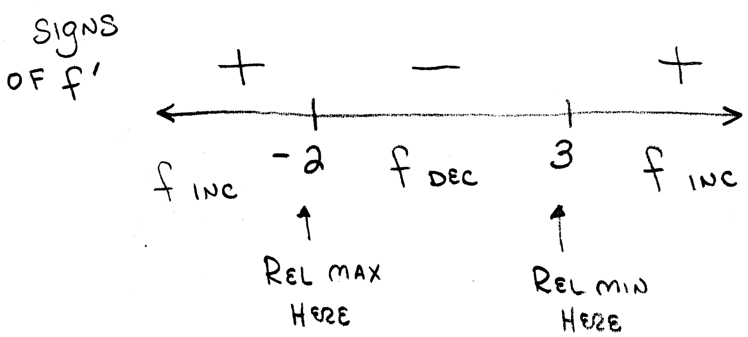
$$35 = 2\pi(50)\frac{dr}{dt}$$

$$\boxed{\frac{dr}{dt} = \frac{35}{100\pi} \approx 0.1114 \text{ m/HR}}$$

10. (12 points) Find open intervals on which the graph of $f(x) = 2x^3 - 3x^2 - 36x + 14$ is increasing/decreasing. Also identify all relative extrema.

$$f'(x) = 6x^2 - 6x - 36$$
$$= 6(x^2 - x - 6)$$
$$= 6(x-3)(x+2) = 0.$$
$$\Rightarrow x = 3, \; x = -2$$

$f'(x)$ DNE NOWHERE



f IS INCREASING ON $(-\infty,-2) \cup (3,\infty)$

f IS DECREASING ON $(-2,3)$

$f(-2) = 58$ IS A REL MAX

$f(3) = -67$ IS A REL MIN

11. (4 points) Which equation explicitly defines a function, and which implicitly defines a function? How can you tell the difference?

$$y = \frac{5x}{x^2+1} \qquad\qquad x^2 + y^2 = 9$$

EXPLICIT

IMPLICIT

y IS EXPLICITLY DESCRIBED IN TERMS OF X:

$y = f(x)$

X'S AND y'S ARE MIXED UP.

y IS NOT EXPLICITLY GIVEN IN TERMS OF y:

$f(x,y) = 9$