

# Math 157 - Test 2

October 23, 2013

Name \_\_\_\_\_

Score \_\_\_\_\_

Show all work to receive full credit. Supply explanations where necessary.

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1. (10 points) Let  $f(x) = 10\sqrt{x}$ .

(a) Find  $f'(x)$ .

(b) Find an equation of the line tangent to the graph of  $f$  at the point where  $x = 4$ .

(c) Find  $f''(x)$ .

2. (4 points) Find  $\frac{d^{17}y}{dx^{17}}$  if  $y = 3x^{10} - 8x^7 - 9x^2 + 12$ .

3. (6 points) The profit, in dollars, from selling  $x$  laptop computers is given by

$$P = -0.04x^2 + 25x - 1500.$$

Find the sales level that results in a marginal profit of zero.

4. (8 points) Suppose  $y$  is implicitly defined as a function of  $x$  by the equation

$$4x^2 = 3y^2 + xy.$$

Find  $dy/dx$  at  $(1, 1)$ .

5. (20 points) Compute the derivative of each function.

(a)  $f(x) = \frac{4}{x} + 9$

(b)  $g(t) = (t^2 - 2t + 5)(t^3 - 1)$

(c)  $y = \frac{x^2 + 6x + 5}{\sqrt[5]{x}}$

(d)  $f(x) = (4x^{-1} - x^4 + 2)^5$

6. (8 points) An object is moving along the graph of  $y^2 - 5x^2 = -1$  in such a way that  $\frac{dx}{dt} = -3$ . Find  $\frac{dy}{dt}$  at the point  $(1, 2)$ .

7. (8 points) A national distributor of pet toys determines the cost and revenue functions for one of its toys:

$$C = 1.2x - 0.0001x^2, \quad 0 \leq x \leq 6000$$

$$R = 3.6x - 0.0005x^2, \quad 0 \leq x \leq 6000$$

Determine the open interval(s) on which the profit is increasing.

8. (12 points) A woman jumps off a diving board and soars through the air in such a way that her height, in feet, at time  $t$  (in seconds) is given by

$$h = -16t^2 + 16t + 32.$$

- (a) Find the diver's velocity function.
- (b) When does the diver reach her highest point?
- (c) What is the diver's average velocity over the first 2 seconds?
- (d) When does the diver hit the water?
9. (8 points) An oil tanker has run aground and ruptured its hull. Leaking oil is spreading in all directions. The polluted region is a circle, which is growing steadily at a rate  $35 \text{ m}^2/\text{hr}$ . How fast is the radius of the oil slick growing at the moment when the radius is  $50 \text{ m}$ ? (The area of a circle is given by  $A = \pi r^2$ .)

10. (12 points) Find open intervals on which the graph of  $f(x) = 2x^3 - 3x^2 - 36x + 14$  is increasing/decreasing. Also identify all relative extrema.

11. (4 points) Which equation explicitly defines a function, and which implicitly defines a function? How can you tell the difference?

$$y = \frac{5x}{x^2 + 1}$$

$$x^2 + y^2 = 9$$