Math 157 - Test 3a

November 20, 2013

Name key

Score ________

Show all work. Supply explanations when necessary.

1. (10 points) Find open intervals on which the graph of $f(x) = x^4 + x^3 - 3x^2 + 1$ is concave up/down. Also identify all points of inflection.

$f'(x) = 4x^3 + 3x^2 - 6x$

$f''(x) = 12x^2 + 6x - 6 = 6(2x^2 + x - 1)$

$= 6(2x-1)(x+1) = 0$

$\Rightarrow x = \frac{1}{2}$ or $x = -1$

Signs of f'': $+$ $-$ $+$

-1, $f(-1) = -2$ $\quad$ $\frac{1}{2}$, $f(\frac{1}{2}) = 7/16$

The graph of f is concave up on $(-\infty, -1) \cup (\frac{1}{2}, \infty)$

Concave down on $(-1, \frac{1}{2})$

$(-1, -2)$ and $(\frac{1}{2}, \frac{7}{16})$ are inflection pts.

2. (5 points) In solving an optimization problem, Joe found that $x = 1$ is a critical number of the function $P(x) = 2x + \dfrac{2}{x}$. Show that Joe's critical number minimizes P.

$\underbrace{P'(x) = 2 - \frac{2}{x^2}}$

Notice that $x = 1$ is a critical number since $P'(1) = 0$.

$P''(x) = \frac{4}{x^3}$

Since $P''(1) > 0$, $x = 1$ must give a minimum.

3. (7 points) The revenue, in dollars, generated by selling x units of a certain product is given by $R(x) = 50xe^{-0.0025x}$. Use differentials to estimate the change in revenue as x changes from 1000 to 1050.

$$R'(x) = 50e^{-0.0025x} + 50xe^{-0.0025x}(-0.0025)$$

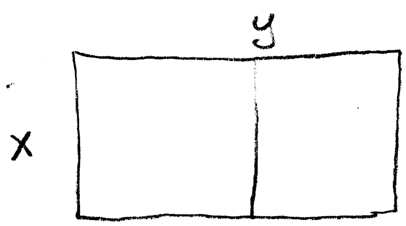
$$dR = R'(x)\,dx \Rightarrow \Delta R \approx R'(x)\,\Delta x$$

$$\begin{matrix} x = 1000 \\ \Delta x = 50 \end{matrix} \Rightarrow \Delta R \approx R'(1000)(50)$$

$$= \boxed{-307.8187}$$

IN COMPARISON, EXACT $\Delta R \approx -301.16$

4. (11 points) A rancher has 200 feet of fencing to enclose two adjacent rectangular corrals (which share one common side). What dimensions should be used so that the enclosed area will be a maximum?



MAXIMIZE $A = xy$

SUBJECT TO

$$3x + 2y = 200$$

$$y = \frac{200 - 3x}{2}$$

$$A(x) = x\left(\frac{200-3x}{2}\right)$$

$$A(x) = \frac{1}{2}(200x - 3x^2), \quad 0 \le x \le \frac{200}{3}$$

$$A'(x) = 100 - 3x = 0$$

$$\Rightarrow x = \frac{100}{3}$$

CHECK CRIT NUMBERS AND ENDPTS...

$$A(0) = 0$$

$$A\left(\frac{200}{3}\right) = 0$$

$$A\left(\frac{100}{3}\right) = \frac{5000}{3} \leftarrow \text{MAX!}$$

DIMENSIONS ARE

$$x = \frac{100}{3} \text{ FT}$$

$$y = 50 \text{ FT}$$

5. (3 points) Use the exponent laws to simplify each expression.

(a) $(5^7)(5^{-3})$ $= 5^{7-3} = \boxed{5^4}$

(b) $(e^3)^x$ $= \boxed{e^{3x}}$

6. (4 points) Use the logarithm laws to expand or condense each expression as appropriate.

(a) $3\log_2 x - 2\log_2(x-1)$ $= \log_2\left[\dfrac{x^3}{(x-1)^2}\right]$

(b) $\ln\left(\dfrac{(x+1)^2}{y-1}\right)$ $= 2\ln(x+1) - \ln(y-1)$

7. (3 points) Find the **exact** value of of each expression.

(a) $\log_6 36$ $= 2$

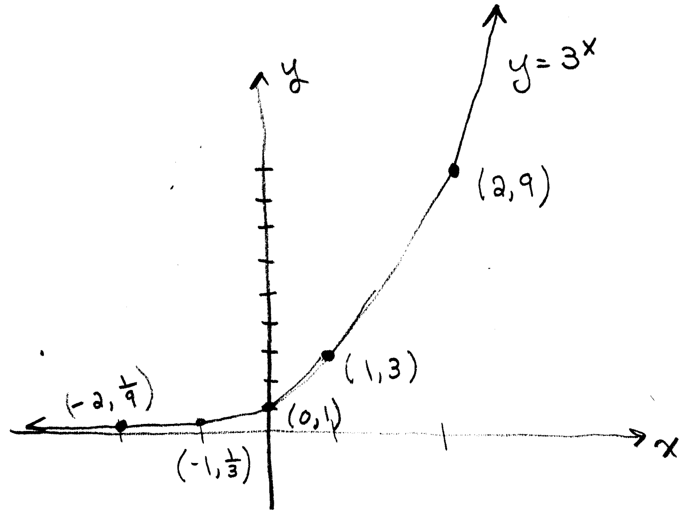
SINCE $6^2 = 36$

(b) $e^{\ln\sqrt{5}}$ $= \sqrt{5}$

(c) $\ln e^{\pi}$ $= \pi$

8. (5 points) Make a table of some values of the function $f(x) = 3^x$. Then plot points to sketch the graph.

x	$f(x) = 3^x$
-2	$\frac{1}{9}$
-1	$\frac{1}{3}$
0	1
1	3
2	9



9. (9 points) Solve for x.

(a) $\ln x^3 = 6$

$$3 \ln x = 6$$

$$\ln x = 2$$

$$\boxed{x = e^2}$$

(b) $4e^{2x-3} - 5 = 0$

$$4e^{2x-3} = 5$$

$$e^{2x-3} = \frac{5}{4}$$

$$2x - 3 = \ln\left(\frac{5}{4}\right)$$

$$\boxed{\begin{array}{l} x = \frac{1}{2}\left(3 + \ln\left(\frac{5}{4}\right)\right) \\ x \approx 1.61157 \end{array}}$$

(c) $\ln x + \ln(x-3) = 0$

$$\ln\left(x(x-3)\right) = 0$$

$$x(x-3) = 1$$

$$x^2 - 3x - 1 = 0$$

$$x = \frac{3 \pm \sqrt{9+4}}{2} = \frac{3 \pm \sqrt{13}}{2}$$

$$\boxed{x = \frac{3+\sqrt{13}}{2} \approx 3.30}$$

OR

$$x = \frac{3-\sqrt{13}}{2} \approx -0.30$$

No good! NOT IN DOMAIN OF $y = \ln x$

10. (6 points) Let $g(x) = 8x^5 - 5x^4 - 20x^3$. Without looking at the graph of g, determine whether the graph is concave up or concave down at the point where $x = 1$.

$$g'(x) = 40x^4 - 20x^3 - 60x^2$$

$$g''(x) = 160x^3 - 60x^2 - 120x$$

$$g''(1) = 160 - 60 - 120 = -20 \Rightarrow \boxed{\text{GRAPH IS CD AT } X = 1}$$

11. (12 points) Determine the derivative of each function. It may be helpful to use exponent laws, logarithm laws, or change-of-base formulas to simplify the functions before differentiation.

(a) $f(x) = (e^x)^2$

$$f(x) = e^{2x} \qquad \boxed{f'(x) = 2e^{2x}}$$

(b) $g(t) = \ln[t^2(t+4)^3]$ $= 2\ln t + 3\ln(t+4)$

$$\boxed{g'(t) = \frac{2}{t} + \frac{3}{t+4}}$$

(c) $f(x) = \log_5 x$ $= \dfrac{\ln x}{\ln 5}$

$$\boxed{f'(x) = \frac{1}{\ln 5}\frac{1}{X}}$$

Math 157 - Test 3b

November 20, 2013

Name key

Score ______

Show all work. Supply explanations when necessary. YOU MUST WORK INDIVIDUALLY ON THIS EXAM.

1. (9 points) The revenue, in dollars, generated by selling x units of a certain product is given by $R(x) = 50xe^{-0.0025x}$. Determine the maximum revenue.

$$R'(x) = 50e^{-0.0025x} + 50xe^{-0.0025x}(-0.0025)$$

$$= 50e^{-0.0025x}(1 - 0.0025x) = 0$$

$$\Rightarrow \quad 1 - 0.0025x = 0$$

$$\Rightarrow \quad x = 400$$

SIGNS OF R': $+$ on $(0, 400)$, $-$ after 400

0 — R IS INC — 400 — R IS DEC

$\boxed{\begin{array}{l} x = 400 \text{ GIVES A MAX} \\ \text{MAX REVENUE IS} \\ R(400) = \$7357.^{59} \end{array}}$

2. (8 points) The circumference of a circle is measured to be 48 cm with an error of ± 0.2 cm. Use differentials to estimate the propagated error in the circle's area.

$$A = \pi r^2$$

$$C = 2\pi r \Rightarrow r = \frac{C}{2\pi}$$

$$A = \pi\left(\frac{C}{2\pi}\right)^2 = \frac{C^2}{4\pi}$$

$$A = \frac{C^2}{4\pi}$$

$$dA = \frac{2C}{4\pi}\,dC$$

$$\Delta A \approx \frac{C}{2\pi}\,\Delta C$$

$$\Delta A \approx \pm \frac{48}{2\pi}(0.2)$$

$$\boxed{\Delta A \approx \pm 1.528 \text{ cm}^2}$$

3. (8 points) The half-life of radioactive radium (^{226}Ra) is 1599 years. How long will it take a sample of 200 g to decay to 20 g? What percent of a given amount will remain after 1000 years?

$$P(t) = P_0 e^{kt}$$

$$\frac{1}{2} = e^{k(1599)} \Rightarrow k = \frac{\ln \frac{1}{2}}{1599}$$

a) $$20 = 200e^{kt} \Rightarrow \frac{1}{10} = e^{kt} \Rightarrow \ln \frac{1}{10} = kt$$

$$t = \frac{\ln \frac{1}{10}}{k} \approx \boxed{5311.76 \text{ YEARS}}$$

b) $$\frac{P_0 e^{1000k}}{P_0} = e^{1000k} = 0.6482\ldots$$

$$\approx \boxed{64.8\%}$$