

Math 157 - Final Exam
December 11, 2013

Name _____

Score _____

Show all work to receive full credit. Supply explanations where necessary.

1. (10 points) Let $f(x) = (x^2 - x + 1)^2$. Find an equation of the line tangent to the graph of f at the point where $x = 2$.

2. (8 points) Find k so that g is continuous at $x = 2$.

$$g(x) = \begin{cases} 8x + k, & x < 2 \\ x^2 + \sqrt{x+2}, & x \geq 2 \end{cases}$$

3. (12 points) Suppose y is implicitly defined as a function of x by the equation

$$6x^2 = y^2 + xy.$$

Find dy/dx at $(1, 2)$.

4. (12 points) Find each limit analytically. Use ∞ , $-\infty$, or DNE if appropriate.

(a) $\lim_{x \rightarrow 8^-} (x^2 - 9x + 7)$

(b) $\lim_{x \rightarrow 2} \frac{x^2 + x - 6}{x^2 - 4}$

(c) $\lim_{x \rightarrow 4^+} \frac{x - 8}{x - 4}$

5. (20 points) Consider the function $f(x) = 2x^3 - 12x^2 + 18x$.

(a) Find all critical numbers of f .

(b) Find open intervals on which f is increasing/decreasing.

(c) Find the relative extreme values of f .

(d) Find open intervals on which the graph of f is concave up/down.

6. (6 points) The profit P (in dollars) for producing x units of a product is given by $P = -2x^2 + 72x - 145$. Find the production level at which the marginal profit is zero.

7. (10 points) Let $f(x) = x^2e^{-2x}$. Find $f''(x)$.

8. (8 points) The revenue for a company selling x units of a product is

$$R = 1200x - 0.15x^2.$$

Use differentials to estimate the change in revenue as the sales increase from 2000 units to 2025 units.

9. (12 points) A farmer plans to fence a rectangular pasture adjacent to a river. The pasture must have an area of 88,200 square meters. No fencing is needed along the river. What dimensions will require the least amount of fencing?

10. (8 points) Determine the derivative of each function. It may be helpful to use exponent laws, logarithm laws, or change-of-base formulas to simplify the functions before differentiation.

(a) $g(t) = \ln[t^3(t + 5)^2]$

(b) $f(x) = \log_2 x$

11. (12 points) Find the area of the region bounded by the graphs of $y = x^2$ and $y = 2x + 3$. You may use your calculator to evaluate your definite integral.

12. (8 points) Evaluate the definite integral: $\int_{-1}^1 3x^2 (x^3 + 1)^4 dx$.

13. (10 points) Evaluate the indefinite integral: $\int \left(6x^2 - \frac{6}{x} + 2e^x \right) dx$

14. (8 points) The half-life of radioactive radium (^{226}Ra) is 1599 years. How long will it take a sample of 150 g to decay to 8 g?

15. (6 points) Use integration by parts to evaluate the indefinite integral: $\int (x^2 \ln x) dx$