Math 157 - Quiz 2

September 2, 2015

Name __key__

Score _____

Show all work to receive full credit. Supply explanations when necessary.

1. (2 points) Solve each equation for t.

   (a) $30 = 100e^{-0.025t}$ $\Rightarrow$ $\dfrac{30}{100} = e^{-0.025t}$ $\Rightarrow$ $\ln 0.3 = -0.025t$

   $$t = \frac{\ln 0.3}{-0.025} \approx 48.159$$

   (b) $8 = 3 + 4e^{2t}$

   $5 = 4e^{2t}$ $\Rightarrow$ $\dfrac{5}{4} = e^{2t}$ $\Rightarrow$ $\ln \dfrac{5}{4} = 2t$

   $$t = \frac{\ln 5/4}{2} \approx 0.112$$

2. (4 points) A quantity is growing so that its annual growth rate is 50%. Suppose that the initial quantity is 80.

   (a) Find a formula that models the growth. That is, find a formula for the quantity as a function of time.

   $P_0 = 80$

   $a - 1 = 0.50 \Rightarrow a = 1.50$

   $$P(t) = 80\,(1.5^t)$$

   (b) Use your function to determine the quantity after 10 years.

   $$P(10) = 80\left(1.5^{10}\right) \approx 4613.2$$

   (c) After how many years will the quantity surpass 1000.

   $1000 = 80\left(1.5^t\right)$ $\Rightarrow$ $\dfrac{100}{8} = 1.5^t$ $\Rightarrow$ $\ln \dfrac{100}{8} = t \ln 1.5$

   $$t = \frac{\ln 100/8}{\ln 1.5} \approx 6.23 \text{ yrs}$$

   (d) The annual growth rate is 50%. Find the equivalent continuous growth rate.

   $e^k = 1.5$

   $k = \ln 1.5 \approx 0.405$

   ABOUT 40.5%

3. (4 points) The antidepressant fluoxetine (Prozac) has a half-life of about 3 days.

(a) What percentage of a dose remains in the body after one day?

$$k = \frac{\ln \frac{1}{2}}{3}$$

$$P(1) = P_0 \, e^{\frac{\ln \frac{1}{2}}{3}} \approx P_0 \, (0.7937)$$

About 79.37%

(b) What percentage of a dose remains in the body after one week?

$$P(7) = P_0 \, e^{\frac{\ln \frac{1}{2}}{3} \cdot 7} \approx P_0 \, (0.1984)$$

About 19.84%