Math 157 - Quiz 7

October 14, 2015

Name __key__

Score _____

Show all work to receive full credit. Supply explanations when necessary.

1. (3 points) Determine the derivative of each function.

(a) $f(x) = \dfrac{1}{(5x^3 + x^2)^{17}} = (5x^3 + x^2)^{-17}$

$$f'(x) = -17(5x^3 + x^2)^{-18}(15x^2 + 2x)$$

$$= \boxed{\dfrac{-17(15x^2 + 2x)}{(5x^3 + x^2)^{18}}}$$

(b) $y = \ln(x - e^{-x})$

$$\dfrac{dy}{dx} = \boxed{\dfrac{1}{x - e^{-x}}(1 + e^{-x})}$$

2. (2 points) Find an equation of the line tangent to the graph of $y = (2x + 1)^4$ at the point where $x = 0$.

POINT: $x = 0$, $y = 1^4 = 1$

$(0, 1)$

SLOPE: $\dfrac{dy}{dx} = 4(2x+1)^3(2)$

$m = \dfrac{dy}{dx}\Big|_{x=0} = 8$

LINE:

$y = 8x + b$

$1 = 8(0) + b \Rightarrow b = 1$

$\boxed{y = 8x + 1}$

3. (2 points) Suppose f is a differentiable function with the properties that $f(1) = 5$ and $f'(1) = -3$. Compute the derivative of $g(x) = \sqrt{f(x)}$ at $x = 1$.

$$g(x) = [f(x)]^{1/2} \cdot$$

$$g'(x) = \frac{1}{2}[f(x)]^{-1/2} f'(x)$$

$$g'(1) = \frac{1}{2}[f(1)]^{-1/2} f'(1) = \frac{1}{2}(5)^{-1/2}(-3) = \boxed{\frac{-3}{2\sqrt{5}}}$$

4. (3 points) Determine the derivative of each function.

(a) $f(t) = t^2 e^{3-5t}$

$$f'(t) = 2t e^{3-5t} + t^2 e^{3-5t}(-5)$$

$$= \boxed{(2t - 5t^2) e^{3-5t}}$$

(b) $y = \sqrt{z}\ln(z^2 + 1) = z^{1/2} \ln(z^2+1)$

$$\frac{dy}{dz} = \boxed{\frac{1}{2} z^{-1/2} \ln(z^2+1) + z^{1/2}\left(\frac{1}{z^2+1}\right)(2z)}$$