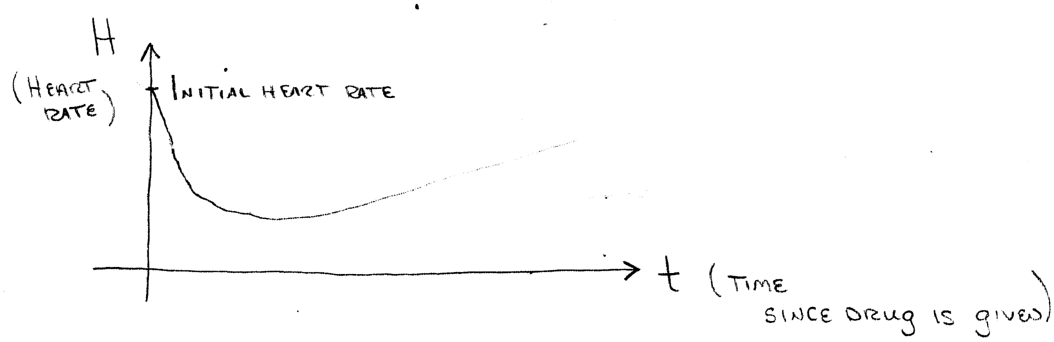
Math 157 - Test 1
September 16, 2015

Name key

Score

Show all work to receive full credit. Supply explanations where necessary.

1. (5 points) When a patient with a rapid heart rate takes a drug, the heart rate plunges dramatically and then slowly rises again as the as the drug wears off. Sketch the graph of a function that might model the patient's heart rate as a function of time. Label your axes, but you need not include a scale.



2. (6 points) Determine the linear function whose graph passes through the points $(4, 5)$ and $(2, -1)$.

$$m = \frac{\Delta y}{\Delta x} = \frac{5-(-1)}{4-2} = \frac{6}{2} = 3$$

$$\boxed{f(x) = 3x - 7}$$

$$f(x) = 3x + b$$

$$f(2) = -1 = 3(2) + b \Rightarrow b = -7$$

3. (8 points) Use algebra to find the limit.

$$\lim_{x \to -3} \frac{x^2 + 8x + 15}{x + 3} \qquad \frac{0}{0}$$

$$= \lim_{x \to -3} \frac{\cancel{(x+3)}(x+5)}{\cancel{x+3}}$$

$$= \lim_{x \to -3} (x+5) = \boxed{2}$$

4. (6 points) Solve for t.

(a) $3e^{5t} = 12e^{2t}$

$$\frac{e^{5t}}{e^{2t}} = \frac{12}{3} \Rightarrow e^{3t} = 4$$

$$3t = \ln 4 \Rightarrow \boxed{t = \frac{\ln 4}{3} \approx 0.462}$$

(b) $113 = 8(1.64)^t$

$$\frac{113}{8} = 1.64^t$$

$$\ln \frac{113}{8} = t \ln 1.64 \Rightarrow \boxed{t = \frac{\ln 113/8}{\ln 1.64} \approx 5.353}$$

5. (5 points) Let $f(x) = -2x^3+4x$. Compute the average rate of change of f from $x = -1$ to $x = 2$.

$$\frac{f(2) - f(-1)}{2-(-1)} = \frac{[-16+8] - [2-4]}{3} = \frac{-6}{3} = \boxed{-2}$$

6. (6 points) Fill in the blank portions of the table below in such a way that the function f is a nonconstant, linear function. Show work or explain. (There are many possible correct answers!)

1 2 3 3

x	1	2	4	7	10
$f(x)$	3	6	12	21	30

3 6 9 9

$f(x) = 3x$

CONSTANT RATE OF CHANGE

$m = \frac{\Delta y}{\Delta x} = 3$

7. (12 points) For his dog-walking services, Joe charges \$8 per hour plus an additional fixed cost of \$7. Monique simply charges \$12 per hour.

(a) For each person, determine the cost function for t hours of service. (These are linear functions.)

JOE: $J(t) = 8t + 7$

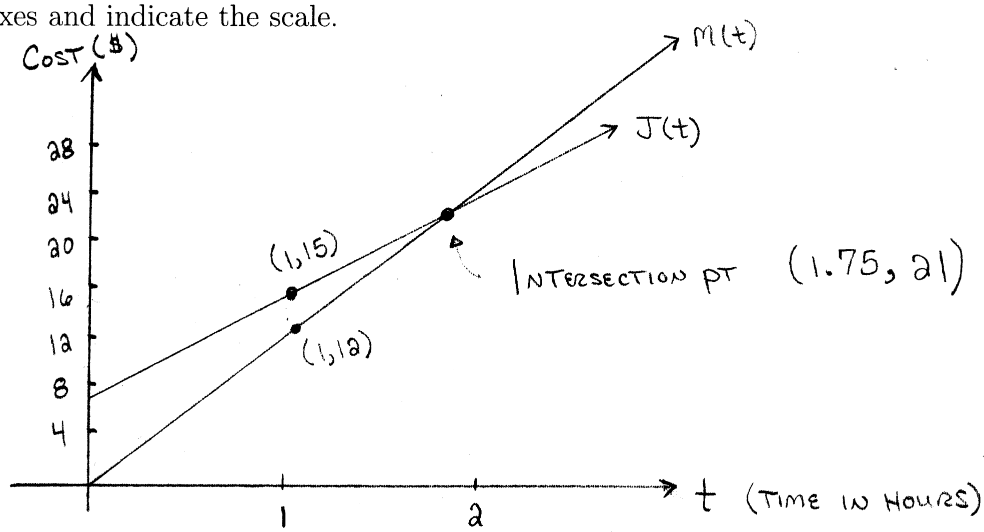
MONIQUE: $M(t) = 12t$

(b) For how many hours of service are the two costs equal?

$8t + 7 = 12t \Rightarrow 7 = 4t$

$\Rightarrow t = \frac{7}{4} = 1.75$ HRS

(c) Graph each cost function and label the point you found in part (b). Label your axes and indicate the scale.



8. (4 points) Use any method to compute or estimate the limit.

$$\lim_{w \to 5} \frac{5 + 10e^{-w}}{w + 5}$$

PLUG IN ... $\lim_{w \to 5} \frac{5 + 10e^{-w}}{w + 5}$

$= \frac{5 + 10e^{-5}}{5 + 5} \approx 0.507$

9. (12 points) A quantity is growing so that its annual growth rate is 20%. Suppose that the initial quantity is 65.

(a) Find a formula that models the growth. That is, find a formula for the quantity as a function of time.

$$P(t) = 65(1.20)^t$$

(b) Use your function to determine the quantity after 25 years.

$$P(25) = 65(1.20)^{25} \approx 6200.754$$

(c) After how many years will the quantity surpass 800?

$$P(t) = 800$$

$$65(1.20)^t = 800$$

$$1.20^t = \frac{800}{65}$$

$$t \ln 1.20 = \ln 800/65$$

$$t = \frac{\ln 800/65}{\ln 1.20}$$

$$t \approx 13.768 \text{ yrs}$$

(d) The annual growth rate is 20%. Find the equivalent continuous growth rate.

$$e^k = 1.20$$

$$k = \ln 1.20 \approx 0.1823$$

18.23%

10. (6 points) The half-life of radioactive strontium–90 is 29 years. In 1960, strontium–90 was released into the atmosphere during the testing of nuclear weapons, and it was absorbed into people's bones.

(a) Find the continuous decay rate. (In class, we called this k.)

$$\frac{\ln \frac{1}{2}}{k} = 29 \quad \Rightarrow \quad k = \frac{\ln \frac{1}{2}}{29} = -0.0239\ldots$$

DECAYS AT A RATE OF 2.39%

(b) How many years does it take until only 10% of the initial amount absorbed remains in the people's bones?

$$0.10 P_0 = P_0 e^{kt}$$

$$0.10 = e^{kt}$$

$$\ln 0.10 = kt$$

$$t = \frac{\ln 0.10}{k} \approx 96.336 \text{ yrs}$$

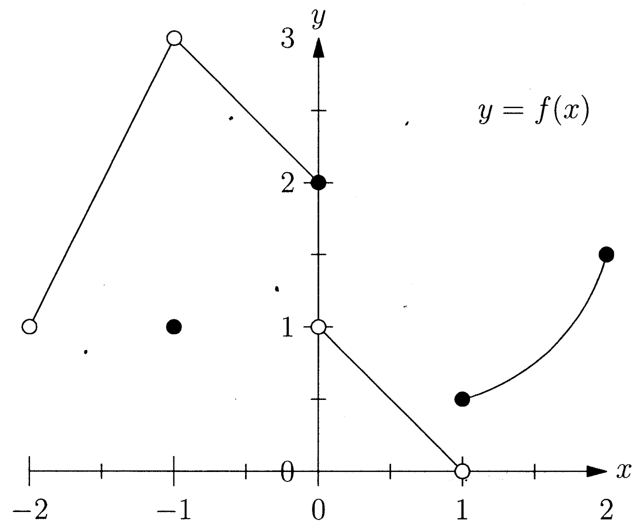
11. (6 points) Use a table of values to estimate the following limit. Your table must show function values at six or more points.

$$\lim_{h \to 0} \frac{13^h - 1}{h} \approx 2.56$$

h	$\frac{13^h - 1}{h}$
0.1	2.9239
0.01	2.5981
0.001	2.5682
0.0001	2.5653
-0.1	2.2624
-0.01	2.5323
-0.001	2.5617

IN FACT, THE LIMIT IS EXACTLY $\ln 13$.

12. (10 points) The graph of the function f is shown below.



(a) On the interval $-2 < x < 2$, at which points is f discontinuous?

$X = -1, \ X = 0, \ X = 1$

(b) Briefly explain why $\lim_{x \to 1} f(x)$ fails to exist.

THE LIMIT FROM THE LEFT IS DIFFERENT FROM THE LIMIT FROM THE RIGHT.

(c) Estimate the value of $\lim_{x \to -1} f(x)$.

$\lim_{X \to -1} f(x) = 3$

(d) Estimate the value of $f(-1)$.

$f(-1) = 1$

(e) Compute the average rate of change over the interval from $x = 1$ to $x = 2$.

$\frac{f(2) - f(1)}{2-1} = \frac{1.5 - 0.5}{1} = \boxed{1}$

13. (9 points) The following functions give the populations of four towns with time t in years.

$$(A) \qquad P = 500(1.35)^t$$
$$(B) \qquad P = 835(1.07)^t$$
$$(C) \qquad P = 140(1.62)^t$$
$$(D) \qquad P = 1150(0.95)^t$$

(a) Which town has the largest percent growth rate? What is that growth rate?

C AT 62%

(b) Which town has the largest initial population? What is that population?

D AT 1150

(c) Which towns (if any) are decreasing in size? Explain.

ONLY D $\quad a = 0.95 < 1 \Rightarrow$ DECREASING AT 5%

14. (5 points) In March 1981, 1st class postage increased from 15 cents to 18 cents. In February 1991, 1st class postage increased from 25 cents to 29 cents. Compute each relative change.

1981: $\dfrac{18-15}{15} = \dfrac{3}{15} = 0.2 = 20\%$

1991: $\dfrac{29-25}{25} = \dfrac{4}{25} = 0.16 = 16\%$