Math 157 - Test 2

October 20, 2015

Name key

Score _____

Show all work. Supply explanations where necessary.

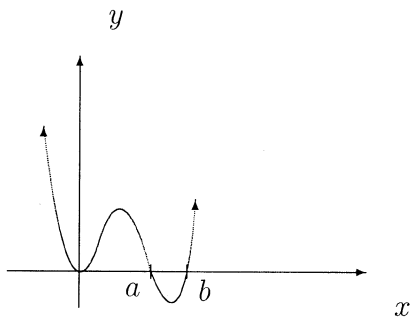
1. (6 points) The quantity (in mg) of a drug in the blood at time t (in minutes) is given by $Q(t) = 25(0.8)^t$. Use a single small interval to estimate the instantaneous rate of change of the quantity at $t = 3$. Give units on your answer.

$[2.99, 3.01] \rightarrow Q'(3) \approx \dfrac{Q(3.01) - Q(2.99)}{3.01 - 2.99}$

$\approx \dfrac{-0.057125}{0.02} \approx -2.9$

$Q'(3) \approx -2.9$ mg/min

2. (6 points) The graph of the function f is shown here. Use the graph to determine whether each the quantities below is positive, negative, or approximately zero. Give a brief explanation for each.



(a) $f'(0) \approx 0$ THE TANGENT LINE AT $X = 0$ IS HORIZONTAL.

(b) $f'(a)$ IS NEGATIVE TANGENT LINE AT $X = a$ SLOPES DOWNWARD.

(c) $f'(b)$ IS POSITIVE TANGENT LINE AT $X = b$ SLOPES UPWARD.

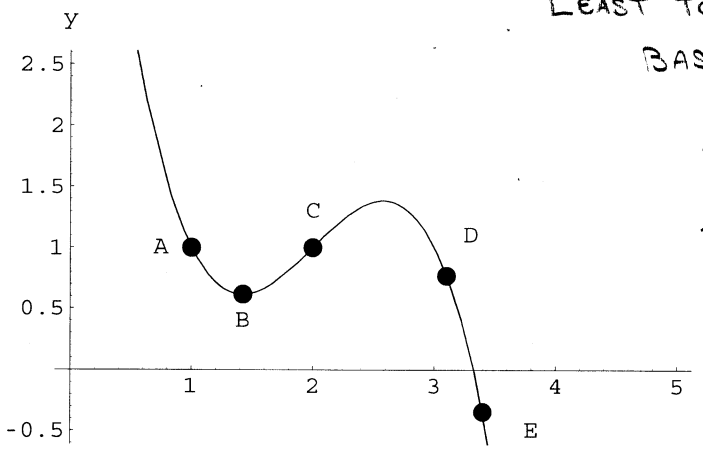
3. (4 points) Let $g(x) = 3x^2 + x$. Use a single small interval to estimate the derivative of g at $x = 1$.

$[0.999, 1.001] \longrightarrow g'(1) \approx \dfrac{g(1.001) - g(0.999)}{1.001 - 0.999} = \dfrac{0.014}{0.002} = 7$

$g'(1) \approx 7$

4. (6 points) The graph of the function f is shown below. Think about the value of the derivative at each of the labeled points. Arrange these values in ascending order.

$$f'(A),\ f'(B),\ f'(C),\ f'(D),\ f'(E)$$



LEAST TO GREATEST BASED ON SLOPES...

$f'(E)$

$f'(D)$

$f'(A)$

$f'(B)$

$f'(C)$

5. (4 points) Refer to the graph of f above.

(a) Choose a labeled point at which $f''(x) > 0$. Briefly explain how you know.

B GRAPH IS CU AT B

(b) Choose a labeled point at which $f''(x) < 0$. Briefly explain how you know.

D GRAPH IS CD AT D

6. (4 points) Joe correctly determined that the derivative of $f(x) = e^{x^2-1}$ is given by $f'(x) = 2xe^{x^2-1}$. Use Joe's work to determine the slope of the line tangent to the graph of f at $x = -1$.

$$m = f'(-1) = 2(-1)e^{(-1)^2-1} = \boxed{-2}$$

7. (4 points) A baked potato has just been taken out of the oven and is cooling off before being eaten. The temperature of the potato (in °F) after t minutes is given by the function $g(t)$.

   (a) Is $g'(t)$ positive or negative? Explain.

$g'(t)$ IS NEGATIVE BECAUSE THE TEMP IS DECREASING.

$g'(t) < 0$

   (b) What are the units on $g'(t)$?

$g'(t)$ IS IN UNITS OF DEGREES PER MINUTE.

8. (8 points) Use the 2nd derivative to determine whether the graph of $y = x^2 + \ln x$ is concave up or concave down at the point where $x = 1$.

$$\frac{dy}{dx} = 2x + \frac{1}{x} = 2x + x^{-1}$$

$$\frac{d^2y}{dx^2} = 2 - x^{-2} = 2 - \frac{1}{x^2}$$

$$\left.\frac{d^2y}{dx^2}\right|_{x=1} = 2 - \frac{1}{1^2} = 1 > 0$$

$\Rightarrow$ GRAPH IS CU.

9. (25 points) Determine the derivative of each function. Use the correct notation when naming your derivative.

(a) $g(x) = \dfrac{4}{x^5} = 4x^{-5}$

$g'(x) = -20x^{-6}$

(b) $y = 6x^5 - 8x^3 + 7x - 10$

$\dfrac{dy}{dx} = 30x^4 - 24x^2 + 7$

(c) $R = (t^2 + 1)\ln t$

$\dfrac{dR}{dt} = 2t \ln t + (t^2+1)\left(\dfrac{1}{t}\right)$

(d) $P = \sqrt{x^2 + 4x - 3} = (x^2+4x-3)^{1/2}$

$\dfrac{dP}{dx} = \dfrac{1}{2}(x^2+4x-3)^{-1/2}(2x+4)$

(e) $f(x) = \dfrac{(x^3 + 2x)^4}{e^{2x}}$

$f'(x) = \dfrac{e^{2x}(4)(x^3+2x)^3(3x^2+2) - (x^3+2x)^4(2e^{2x})}{e^{4x}}$

10. (7 points) Find an equation of the line tangent to the graph of $h(x) = e^{-5x}$ at the point where $x = 0$.

Point: $x = 0$, $y = h(0) = 1$

$(0, 1)$

Slope: $h'(x) = -5e^{-5x}$

$m = h'(0) = -5$

Line:

$y = -5x + b$

$1 = -5(0) + b \Rightarrow b = 1$

$\boxed{y = -5x + 1}$

11. (10 points) The table below gives the values of the functions f and g and their derivatives at selected values of x.

x	-2	-1	2
$f(x)$	1	3	-2
$f'(x)$	2	-1	-1
$g(x)$	2	0	-2
$g'(x)$	-3	-2	1

(a) If $h(x) = f(g(x))$, use the chain rule to compute $h'(2)$.

$h'(x) = f'(g(x))\, g'(x)$

$h'(2) = f'(g(2))\, g'(2) = f'(-2)\,(1)$

$= (2)(1) = \boxed{2}$

(b) If $h(x) = f(x) \cdot g(x)$, use the product rule to compute $h'(-1)$.

$h'(x) = f'(x)g(x) + f(x)g'(x)$

$h'(-1) = f'(-1)g(-1) + f(-1)g'(-1)$

$= (-1)(0) + (3)(-2) = \boxed{-6}$

12. (4 points) The function $g(x)$ is a linear function whose graph passes through the origin. Determine a formula for the function g if $g'(3) = -7$.

$g(x) = mx$

$g'(x) = m$

$g'(3) = -7 \Rightarrow m = -7$

$g(x) = -7x$

13. (8 points) With t in years since January 1, 2010, the population P of Slim Chance is predicted by

$$P = 35000(0.98)^t.$$

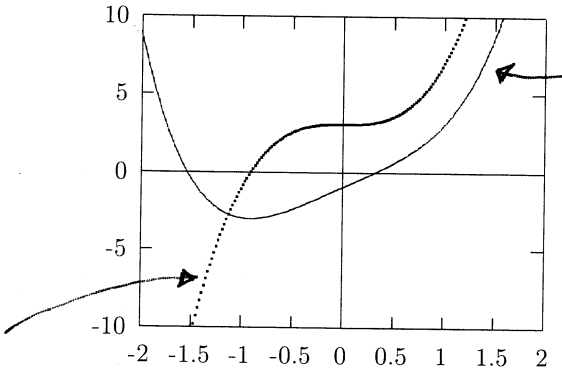
At what rate will the population be changing on January 1, 2023?

$$\begin{array}{r} 2023 \\ -\ 2010 \\ \hline 13 \end{array}$$

$$\frac{dP}{dt} = 35000\,(\ln 0.98)\,(0.98)^t$$

$$\left.\frac{dP}{dt}\right|_{t=13} = -543.8 \text{ people per year}$$

14. (4 points) The following figure shows the graph of a function and its derivative. Which is which? Give at least one reason to support your conclusion. (Hint: Think about what derivatives tell us about increasing/decreasing functions.)



FUNCTION

DERIVATIVE

- WHERE THE FUNCTION IS INCREASING/DECREASING, THE DERIVATIVE IS POS/NEG.
- THE DERIVATIVE IS ZERO WHERE THE FUNC'S GRAPH HAS A HORIZONTAL TAN LINE.