Math 157 - Test 3

November 18, 2015

Name key

Score ________

Show all work. Supply explanations where necessary.

1. (12 points) Let $f(x) = 3x^4 - 4x^3$.

   (a) Find open intervals on which f is increasing/decreasing.

$f'(x) = 12x^3 - 12x^2$
$= 12x^2(x-1)$

CRIT PTS ARE
$x = 0,\ x = 1$

SIGNS OF f': $-$, $-$, $+$

0 , 1

f: DEC , DEC , INC

f IS DECREASING ON $(-\infty, 0) \cup (0, 1)$ AND INCREASING ON $(1, \infty)$

   (b) Identify all relative (local) extreme values.

$f(1) = -1$ IS A REL MIN

$f(0) = 0$ IS NEITHER A MAX NOR A MIN

   (c) Find open intervals on which the graph of f is concave up/down.

$f''(x) = 36x^2 - 24x$
$= 12x(3x-2)$

POSSIBLE INF PTS:
$x = 0,\ x = \frac{2}{3}$

SIGNS OF f'': $+$, $-$, $+$

0 , $\frac{2}{3}$

CU , CD , CU

GRAPH IS CU ON $(-\infty, 0) \cup (\frac{2}{3}, \infty)$ AND CD ON $(0, \frac{2}{3})$

   (d) Find all points of inflection.

$x = 0 \Rightarrow y = 0$

$x = \frac{2}{3} \Rightarrow y = -\frac{16}{27}$

CONCAVITY CHANGES AT $(0,0)$ & $\left(\frac{2}{3}, -\frac{16}{27}\right)$

2. (6 points) Let $h(x) = x^4 - 4x^3 + 8x$. Show that h has a critical point at $x = 1$. Then use the 2nd derivative to determine whether there is a relative (local) maximum or minimum at $x = 1$.

$h'(x) = 4x^3 - 12x^2 + 8$

$h'(1) = 4 - 12 + 8 = 0$

$\Rightarrow$ $x = 1$ IS A CRIT PT.

$h''(x) = 12x^2 - 24x$

$h''(1) = 12 - 24 = -12 < 0$ CD

$\boxed{x = 1 \text{ GIVES A REL MAX}}$

3. (8 points) Find the absolute (global) extreme values of $g(x) = 2x^3 - 21x^2 + 60x - 30$ on the interval $0 \le x \le 6$.

$g'(x) = 6x^2 - 42x + 60$

$= 6(x^2 - 7x + 10)$

$= 6(x-5)(x-2) = 0$

$x = 5,\ x = 2$

END PTS: $x = 0,\ x = 6$

CRIT PTS: $x = 2,\ x = 5$

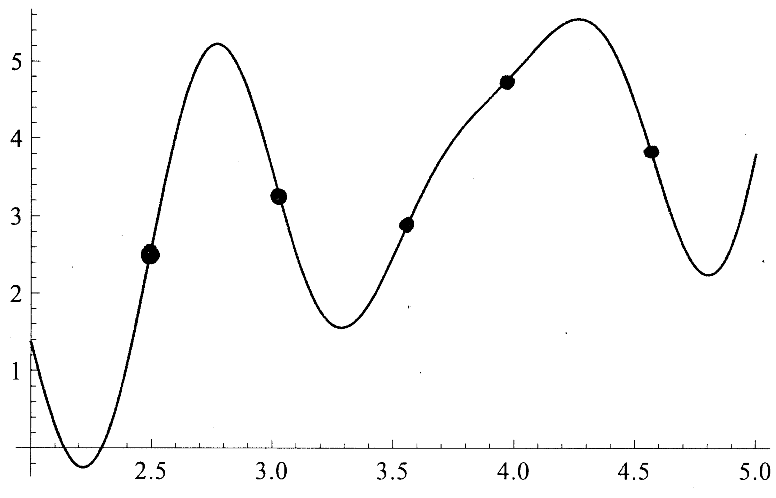
$g(0) = -30$ ← ABS MIN

$g(2) = 22$ ← ABS MAX

$g(5) = -5$

$g(6) = 6$

4. (4 points) Identify any points of inflection on the graph below.



Computed by Wolfram|Alpha

CONCAVITY CHANGES WHERE INDICATED

5. (8 points) The revenue from selling q items is $R(q) = 450q$ and the total cost is $C(q) = 10000 + 3q^2$. Find the quantity that maximizes profit. What is the total profit at that production level? Explain or show that you have indeed found a global maximum.

$P(q) = R(q) - C(q) = 450q - 10000 - 3q^2$

$P'(q) = 450 - 6q = 0$

$\Rightarrow \boxed{q = \frac{450}{6} = 75}$

Signs of P': $+$ on $(0, 75)$, $-$ after 75

$q = 75$ gives a max

$\boxed{P(75) = 6875}$

6. (8 points) The energy expended by a bird per day, E, depends on the time spent foraging for food per day, F hours, according to the equation

$$E = 0.25F + \frac{1.7}{F^2}.$$

Find the foraging time that minimizes the energy expenditure. Explain or show that you have indeed found a global minimum.

$E' = 0.25 - \frac{3.4}{F^3} = 0$

$\Rightarrow F^3 = \frac{3.4}{0.25} = 13.6$

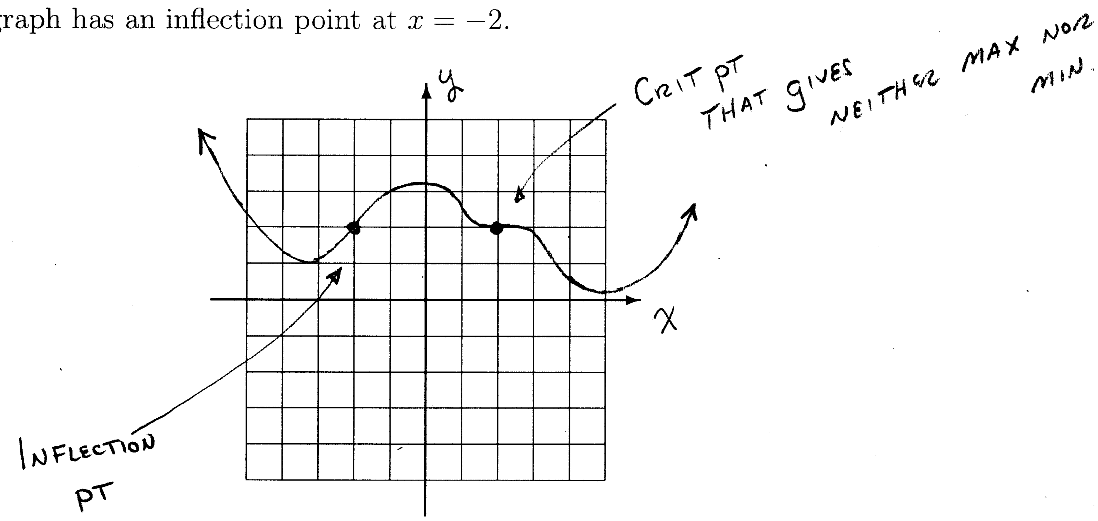
$F \approx 2.387$

Signs of E': $-$ on $(0, \sqrt[3]{13.6})$, $+$ after $\sqrt[3]{13.6}$

$F = \sqrt[3]{13.6} \approx 2.387$ HRS gives an ABS MIN

7. (4 points) Sketch the graph of a function f satisfying

- f has a critical point at $x = 2$, but $f(2)$ is neither a maximum nor a minimum, and
- the graph has an inflection point at $x = -2$.



8. (10 points) A car initially going 50 ft/sec brakes and slows at a constant rate so that it comes to a stop in 5 seconds. Its velocity over the 5-second interval is given by

$$V(t) = 50 - 10t,$$

where t is in seconds and $V(t)$ is in ft/sec.

(a) Make a table of values showing the velocity at $t = 0, 1, 2, 3, 4, 5$.

t	0	1	2	3	4	5
$V(t)$	50	40	30	20	10	0

(b) Use a left sum to estimate the distance traveled by the car over the 5-sec interval.

LEFT SUM $= 50(1) + 40(1) + 30(1) + 20(1) + 10(1)$

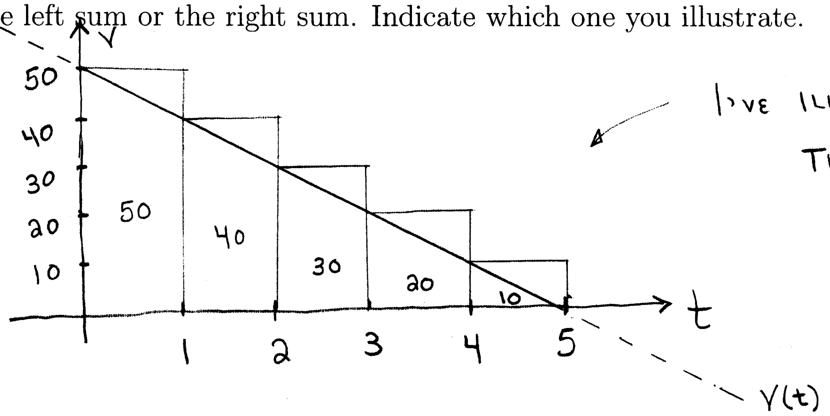
$= \boxed{150 \text{ FT}}$

(c) Use a right sum to estimate the distance traveled by the car over the 5-sec interval.

RIGHT SUM $= 40(1) + 30(1) + 20(1) + 10(1) + 0(1)$

$= \boxed{100 \text{ FT}}$

(d) Sketch the graph of V over the 5-sec interval. Then use rectangles to illustrate either the left sum or the right sum. Indicate which one you illustrate.



I'VE ILLUSTRATED THE LEFT SUM.

9. (6 points) Let $C(q)$ and $R(q)$ represent the cost and revenue functions, in dollars, associated with producing q items. If $C'(50) = 71$ and $R'(50) = 68$, approximately how much profit is earned by the 51st item? At the level $q = 50$, should production be increased or decreased?

$P'(50) = R'(50) - C'(50) = 68 - 71 = -3$

ABOUT \$3 IS LOST!

PROFIT IS DECREASING

$\Rightarrow$ DECREASE PRODUCTION TO INCREASE PROFIT.

10. (8 points) The revenue from selling q items is $R(q) = 30q + 5$ and the total cost is $C(q) = 0.01q^3 - 0.7q^2 + 34q + 8$.

(a) Determine the profit function $P(q)$.

$$P(q) = R(q) - C(q) = [30q + 5] - [0.01q^3 - 0.7q^2 + 34q + 8]$$

$$\boxed{P(q) = -0.01q^3 + 0.7q^2 - 4q - 3}$$

(b) Determine the marginal profit.

$$\boxed{P'(q) = -0.03q^2 + 1.4q - 4}$$

(c) Determine the marginal profit at $q = 25$. Based on your value should you increase or decrease production in order to increase profit?

$P'(25) = 12.25 \Rightarrow$ PROFIT IS INCREASING.

YOU SHOULD INCREASE PRODUCTION.

11. (6 points) Find the critical points of $g(x) = \sqrt[3]{x} - 3x$.

$$g'(x) = \frac{1}{3}x^{-2/3} - 3$$

$g'(x)$ DNE WHEN $X = 0$

$g'(x) = 0$ WHEN

$$\frac{1}{3}x^{-2/3} = 3$$

$$x^{-2/3} = 9$$

$$x^{-1/3} = \pm 3$$

$$x^{1/3} = \pm\frac{1}{3}$$

$$x = \left(\pm\frac{1}{3}\right)^3$$

$$x = \pm\frac{1}{27}$$

CRIT PTS:

$X = 0$

$X = \frac{1}{27}$

$X = -\frac{1}{27}$

12. (12 points) The rate of change of a quantity is given by $f(t) = t^2 + 1$.

(a) Estimate the total (accumulated) change over the interval from $t = 0$ to $t = 8$ by using a left sum with subintervals of length $\Delta t = 2$.

$$\text{LEFT SUM} = f(0)(2) + f(2)(2) + f(4)(2) + f(6)(2)$$
$$= 2(1 + 5 + 17 + 37) = \boxed{120}$$

(b) Estimate the total (accumulated) change over the interval from $t = 0$ to $t = 8$ by using a left sum with subintervals of length $\Delta t = 1$.

$$\text{LEFT SUM} = f(0)(1) + f(1)(1) + \cdots + f(7)(1)$$
$$= 1(1 + 2 + 5 + 10 + 17 + 26 + 37 + 50)$$
$$= \boxed{148}$$

(c) Which one of your results above do you think better estimates the total change? Why?

(b) --- SMALLER SUBINTERVALS SHOULD PROVIDE A BETTER ESTIMATE.

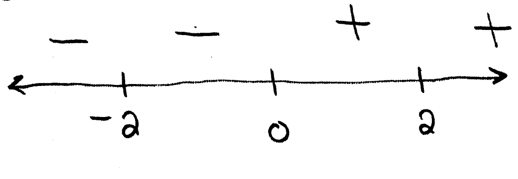
13. (8 points) Let $f(x) = (x^2 - 4)^7$. Find the critical points of f and determine whether they give relative (local) maximums or minimums.

$$f'(x) = 7(x^2 - 4)^6(2x) = 0$$

$x^2 - 4 = 0$ $\quad$ $x = 0$

$x = \pm 2$

SIGNS OF f'



$f(0)$ IS A REL MIN

$f(-2)$ AND $f(2)$ ARE NEITHER MAX NOR MIN