

Math 157 - Test 3

November 18, 2015

Name _____

Score _____

Show all work. Supply explanations where necessary.

1. (12 points) Let $f(x) = 3x^4 - 4x^3$.

(a) Find open intervals on which f is increasing/decreasing.

(b) Identify all relative (local) extreme values.

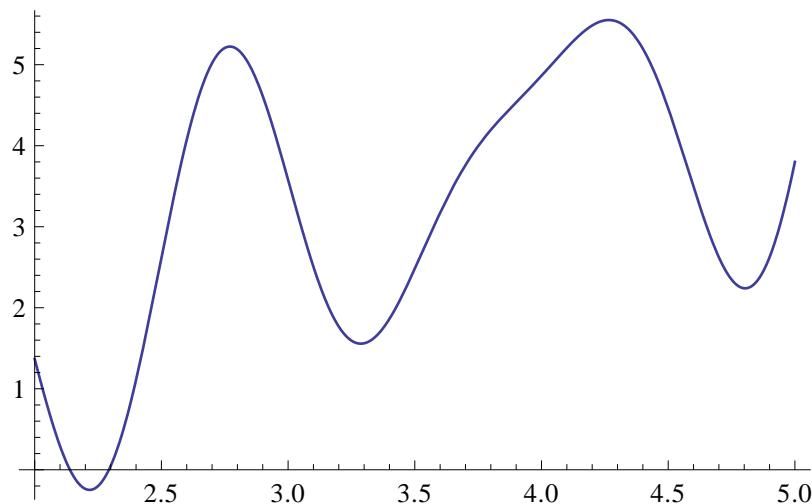
(c) Find open intervals on which the graph of f is concave up/down.

(d) Find all points of inflection.

2. (6 points) Let $h(x) = x^4 - 4x^3 + 8x$. Show that h has a critical point at $x = 1$. Then use the 2nd derivative to determine whether there is a relative (local) maximum or minimum at $x = 1$.

3. (8 points) Find the absolute (global) extreme values of $g(x) = 2x^3 - 21x^2 + 60x - 30$ on the interval $0 \leq x \leq 6$.

4. (4 points) Identify any points of inflection on the graph below.



Computed by Wolfram|Alpha

5. (8 points) The revenue from selling q items is $R(q) = 450q$ and the total cost is $C(q) = 10000 + 3q^2$. Find the quantity that maximizes profit. What is the total profit at that production level? Explain or show that you have indeed found a global maximum.

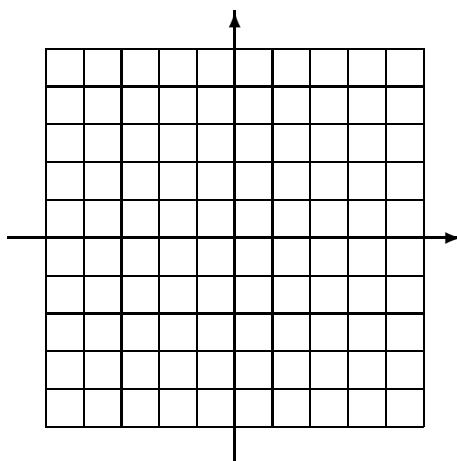
6. (8 points) The energy expended by a bird per day, E , depends on the time spent foraging for food per day, F hours, according to the equation

$$E = 0.25F + \frac{1.7}{F^2}.$$

Find the foraging time that minimizes the energy expenditure. Explain or show that you have indeed found a global minimum.

7. (4 points) Sketch the graph of a function f satisfying

- f has a critical point at $x = 2$, but $f(2)$ is neither a maximum nor a minimum, and
- the graph has an inflection point at $x = -2$.



8. (10 points) A car initially going 50 ft/sec brakes and slows at a constant rate so that it comes to a stop in 5 seconds. Its velocity over the 5-second interval is given by

$$V(t) = 50 - 10t,$$

where t is in seconds and $V(t)$ is in ft/sec.

- (a) Make a table of values showing the velocity at $t = 0, 1, 2, 3, 4, 5$.
- (b) Use a left sum to estimate the distance traveled by the car over the 5-sec interval.
- (c) Use a right sum to estimate the distance traveled by the car over the 5-sec interval.
- (d) Sketch the graph of V over the 5-sec interval. Then use rectangles to illustrate either the left sum or the right sum. Indicate which one you illustrate.

9. (6 points) Let $C(q)$ and $R(q)$ represent the cost and revenue functions, in dollars, associated with producing q items. If $C'(50) = 71$ and $R'(50) = 68$, approximately how much profit is earned by the 51st item? At the level $q = 50$, should production be increased or decreased?

10. (8 points) The revenue from selling q items is $R(q) = 30q + 5$ and the total cost is $C(q) = 0.01q^3 - 0.7q^2 + 34q + 8$.

(a) Determine the profit function $P(q)$.

(b) Determine the marginal profit.

(c) Determine the marginal profit at $q = 25$. Based on your value should you increase or decrease production in order to increase profit?

11. (6 points) Find the critical points of $g(x) = \sqrt[3]{x} - 3x$.

12. (12 points) The rate of change of a quantity is given by $f(t) = t^2 + 1$.
- Estimate the total (accumulated) change over the interval from $t = 0$ to $t = 8$ by using a left sum with subintervals of length $\Delta t = 2$.
 - Estimate the total (accumulated) change over the interval from $t = 0$ to $t = 8$ by using a left sum with subintervals of length $\Delta t = 1$.
 - Which one of your results above do you think better estimates the total change? Why?
13. (8 points) Let $f(x) = (x^2 - 4)^7$. Find the critical points of f and determine whether they give relative (local) maximums or minimums.