

Math 171 - Test 2
October 18, 2012

Name _____

Score _____

Show all work to receive full credit. Supply explanations where necessary.

1. (6 points) A particle is moving along the graph of $x^2y = 12$ in such a way that $\frac{dx}{dt} = 8$.
Find $\frac{dy}{dt}$ when $x = 2$.

2. (8 points) Find the critical numbers of $g(x) = x^{2/3}(x^2 - 4)$.

3. (12 points) An object is launched upward with an initial speed of 80 ft/s over the side of a 96-ft building.

(a) Find a formula for the object's height at time t . Use $g = 32 \text{ ft/s}^2$.

(b) What is the object's average velocity over the first 3 seconds of travel?

(c) Determine the object's velocity function.

(d) Determine the maximum height of the object.

(e) When will the object hit the ground?

(f) Determine the object's acceleration function.

4. (8 points) Find an equation of the line tangent to the graph of $x^2y + y^3 = 2x^2$ at the point $(-1, 1)$.

5. (9 points) An oil tanker has run aground and ruptured its hull. Leaking oil is spreading in all directions. The polluted region is circular and growing steadily at a rate $35 \text{ m}^2/\text{hr}$. How fast is the radius of the oil slick growing at the moment when the radius is 50 m ?

6. (20 points) Differentiate. Do not simplify.

(a) $\frac{d}{dt} \left(6t^4 + \frac{8}{t^2} - \sqrt[3]{t^2} \right)$

(b) $\frac{d}{dx} [5 \cdot \tan(x) \cdot \sec(x)]$

(c) $\frac{d}{dr} \csc(2\pi r^2)$

(d) $\frac{d}{dx} [x^2 \sqrt{x^6 + 7}]$

7. (8 points) The following table gives information about the functions f and g .

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
0	-7	0	4	2
1	-8	-1	3	5
2	-7	4	DNE	DNE
3	2	15	1	5

(a) Use the information to find the derivative of $f(x)g(x)$ (product) at the point where $x = 1$.

(b) Use the information to find the derivative of $f(g(x))$ (composition) at the point where $x = 1$.

8. (4 points) What is the difference between an explicitly-defined function and an implicitly-defined function? Give an example of each.

9. (4 points) Use the quotient rule to derive the formula for the derivative of $y = \cot x$.

10. (7 points) Find the absolute extreme values of $f(x) = x^3 + 3x^2 + 1$ on $[-1, 2]$.

11. (5 points) Determine $\frac{d^3}{dx^3} \sqrt{x}$.

12. (9 points) Gravel is being dumped from a conveyor belt onto a conical pile in such a way that the volume of the pile is increasing at a rate of $30 \text{ ft}^3/\text{min}$. As the gravel pile grows, the base diameter and the height of the conical pile are always equal. How fast is the height of the pile increasing at the moment when the pile is 10 ft tall?

Variables:

Given information and what to find:

Equation(s) relating the variables:

Equation(s) relating the rates:

Solution:

Follow-up question: Does the height of the pile grow more quickly when the pile is small or when the pile is big? Explain your reasoning.