Math 171 - Quiz 11

November 28, 2018

Name key

Score ________

Show all work to receive full credit. Supply explanations when necessary.

1. (3 points) Consider the function $f(x) = x^3 - 16x + 32$, and suppose you wish to find a solution of $f(x) = 0$.

   (a) Use Newton's method starting with $x_0 = 2$. Take several steps. What do you notice?

$f(x) = x^3 - 16x + 32$

$f'(x) = 3x^2 - 16$

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$x_0 = 2$
$x_1 = 4$
$x_2 = 3$
$x_3 = 2$
$x_4 = 4$
$x_5 = 3$
$x_6 = 2$

NEWTON'S METHOD CONTINUALLY CYCLES THROUGH 2, 4, 3.

   (b) Use Newton's method with a better initial guess. Take enough steps to determine the solution with several digits of accuracy.

$x_0 = -5$
$x_1 = -4.77966$
$x_2 = -4.76600$
$x_3 = -4.76595$
$x_4 = -4.76595$ ← THIS APPEAR TO BE ACCURATE TO ALL FIVE DECIMAL PLACES.

2. (3 points) Use differentials to approximate the value of $\sqrt[4]{626}$. Do this entirely without using your calculator.

$f(x) = \sqrt[4]{x} = x^{1/4}$

$\Delta y \approx \frac{1}{4} x^{-3/4} \Delta x$

USE $x = 625$ AND $\Delta x = 1$

$\Delta y = f(x + \Delta x) - f(x)$

$\frac{1}{4}(625)^{-3/4}(1) \approx (626)^{1/4} - (625)^{1/4}$

$\frac{1}{4} \cdot \frac{1}{5^3} + 5 \approx (626)^{1/4}$

$\underbrace{\frac{1}{500} + 5}_{5.002} \approx (626)^{1/4} \Rightarrow (626)^{1/4} \approx \boxed{5.002}$

Follow-up: Now use your calculator to compare your approximation to the calculator's value of $\sqrt[4]{626}$.

$\underbrace{(626)^{1/4} \approx 5.001998801}_{\text{PRETTY GOOD APPROX!}}$

3. (2 points) Evaluate each indefinite integral.

(a) $\int \frac{1}{x\sqrt{x}}\,dx = \int x^{-3/2}\,dx$

$= \frac{x^{-1/2}}{-1/2} + C = -2x^{-1/2} + C$

$= \frac{-2}{\sqrt{x}} + C$

(b) $\int (4x - \csc^2 x)\,dx$

$= 2x^2 + \cot x + C$

4. (2 points) Determine the function $f(s)$ if $f'(s) = 10s - 12s^3$ and $f(3) = 2$.

$f(s) = 5s^2 - 3s^4 + C$

$f(3) = 2 \Rightarrow$

$5(3)^2 - 3(3)^4 + C = 2$

$45 - 243 + C = 2$

$\Rightarrow C = 200$

$f(s) = 5s^2 - 3s^4 + 200$