Math 171 - Quiz 4 Name: key

September 12, 2018 Score ________

Show all work to receive full credit. Supply explanations when necessary.

1. (4 points) Consider the following piecewise function: $g(x) = \begin{cases} 3x^2 + ax, & x \le 1 \\ 5 + b\cos \pi x, & x > 1 \end{cases}$

   (a) Find a so that $\lim_{x\to 1^-} g(x) = 6$.

   $$6 = \lim_{x\to 1^-} g(x) = \lim_{x\to 1} (3x^2 + ax) = 3 + a \Rightarrow 3 + a = 6 \Rightarrow \boxed{a = 3}$$

   (b) Find b so that $\lim_{x\to 4^+} g(x) = 13$.

   $$13 = \lim_{x\to 4^+} g(x) = \lim_{x\to 4} (5 + b\cos \pi x) = 5 + b\cos 4\pi = 5 + b$$
   $$\Rightarrow 13 = 5 + b \Rightarrow \boxed{b = 8}$$

   (c) Find all possible a and b so that $\lim_{x\to 1} g(x)$ exists.

   $$\lim_{x\to 1^-} g(x) = \lim_{x\to 1^+} g(x) \Rightarrow 3 + a = 5 + b\cos \pi$$
   or
   $$3 + a = 5 - b \Rightarrow a + b = 2$$

   Any a, b will work as long as $a + b = 2$.

2. (2 points) Explain why $\lim_{x\to 2} \sqrt{x-2}$ does not exist.

   $\lim_{x\to 2^+} \sqrt{x-2} = 0$, but $\lim_{x\to 2^-} \sqrt{x-2}$ DNE because $\sqrt{x-2}$ is not defined for $x < 2$.

3. (2 points) Evaluate the limit: $\lim_{x\to 5^+} \dfrac{x^2 - 2x - 15}{x^2 - 25}$ $\frac{0}{0}$

   $$\lim_{x\to 5} \frac{x^2 - 2x - 15}{x^2 - 25} = \lim_{x\to 5} \frac{(x-5)(x+3)}{(x-5)(x+5)} = \frac{8}{10} = \boxed{\frac{4}{5}}$$

   Since the two-sided limit exists, the one-sided does and is equal.

4. (2 points) Sketch the graph of a function f such that

   - $\lim_{x\to 2^-} f(x) = 3$
   - $\lim_{x\to 2} f(x)$ exists
   - $f(2) = 0$