Math 171 - Quiz 9

October 31, 2018

Name: key

Score ________

Show all work to receive full credit. Supply explanations when necessary.

1. (8 points) Consider the function $f(x) = 6x^4 + 2x^3 - 12x^2 + 3$.
   - Find open intervals on which f is increasing/decreasing.
   - Identify all relative extreme values.
   - Find open intervals on which the graph of f is concave up/down.
   - Find all inflection points of f.

$$f'(x) = 24x^3 + 6x^2 - 24x$$
$$= 6x(4x^2 + x - 4) = 0$$
$$x = 0, \quad x = \frac{-1 \pm \sqrt{1 + 4(4)(4)}}{8}$$
$$= \frac{-1 \pm \sqrt{65}}{8}$$

Signs of f': $-$ $+$ $-$ $+$

$\frac{-1-\sqrt{65}}{8}$ $\quad 0 \quad$ $\frac{-1+\sqrt{65}}{8}$

f is DEC INC DEC INC

f is decreasing on $\left(-\infty, \frac{-1-\sqrt{65}}{8}\right) \cup \left(0, \frac{-1+\sqrt{65}}{8}\right)$.

f is increasing on $\left(\frac{-1-\sqrt{65}}{8}, 0\right) \cup \left(\frac{-1+\sqrt{65}}{8}, \infty\right)$.

$f\left(\frac{-1-\sqrt{65}}{8}\right) \approx -5.4260$ is a rel min

$f(0) = 3$ is a rel max

$f\left(\frac{-1+\sqrt{65}}{8}\right) \approx -1.3318$ is a rel min

$$f''(x) = 72x^2 + 12x - 24$$
$$= 12(6x^2 + x - 2)$$
$$= 12(3x + 2)(2x - 1) = 0$$
$$x = -\frac{2}{3}, \quad x = \frac{1}{2}$$

Signs of f'': $+$ $-$ $+$

$-\frac{2}{3}$ $\quad$ $\frac{1}{2}$

Graph is CU CD CU

Graph is concave up on $\left(-\infty, -\frac{2}{3}\right) \cup \left(\frac{1}{2}, \infty\right)$.

Graph is concave down on $\left(-\frac{2}{3}, \frac{1}{2}\right)$.

$\left(-\frac{2}{3}, -\frac{47}{27}\right)$ & $\left(\frac{1}{2}, \frac{5}{8}\right)$ are inflection points.

TURN OVER.

2. (2 points) The table below gives the values of a function f and it first two derivatives at selected values of x. Determine which row gives the data for f, which row gives the data for f', and which row gives the data for f''. Explain your reasoning.

x	0.00	0.33	0.66	1.00	1.33	1.66	2.00	2.33	2.66	3.00
$A(x)$	0.00	0.64	1.14	1.38	1.28	0.84	0.08	−0.89	−1.91	−2.83
$B(x)$	0.00	0.11	0.41	0.84	1.30	1.66	1.82	1.69	1.22	0.42
$C(x)$	2.00	1.78	1.16	0.24	−0.83	−1.85	−2.65	−3.07	−3.00	−2.40

$B(x) = f(x)$

$A(x) = f'(x)$

$C(x) = f''(x)$

WHEN B IS INCREASING (ON $(0,2)$), A IS POSITIVE. WHEN B IS DECREASING (ON $(2,3)$), A IS NEGATIVE. THIS SUPPORTS $B'(x) = A(x)$.

WHEN A IS DECREASING (ON $(0,1)$), C IS POSITIVE. WHEN A IS DECREASING (ON $(1,3)$), C IS NEGATIVE. THIS SUPPORTS $A'(x) = C(x)$.