

# Math 171 - Quiz 9

October 31, 2018

Name key

Score \_\_\_\_\_

Show all work to receive full credit. Supply explanations when necessary.

1. (8 points) Consider the function  $f(x) = 6x^4 + 2x^3 - 12x^2 + 3$ .

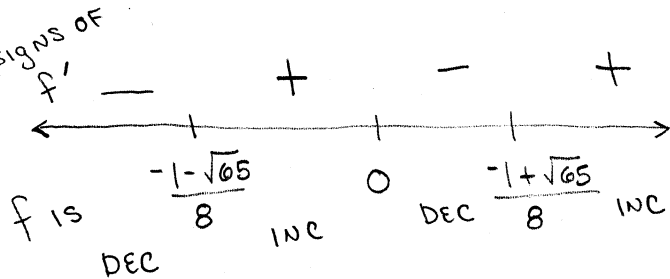
- Find open intervals on which  $f$  is increasing/decreasing.
- Identify all relative extreme values.
- Find open intervals on which the graph of  $f$  is concave up/down.
- Find all inflection points of  $f$ .

$$f'(x) = 24x^3 + 6x^2 - 24x$$

$$= 6x(4x^2 + x - 4) = 0$$

$$x = 0, \quad x = \frac{-1 \pm \sqrt{1 + 4(4)(4)}}{8}$$

$$= \frac{-1 \pm \sqrt{65}}{8}$$



$f$  IS DECREASING ON  $(-\infty, \frac{-1-\sqrt{65}}{8}) \cup (0, \frac{-1+\sqrt{65}}{8})$ .

$f$  IS INCREASING ON  $(\frac{-1-\sqrt{65}}{8}, 0) \cup (\frac{-1+\sqrt{65}}{8}, \infty)$ .

$f(\frac{-1-\sqrt{65}}{8}) \approx -5.4260$  IS A REL MIN

$f(0) = 3$  IS A REL MAX

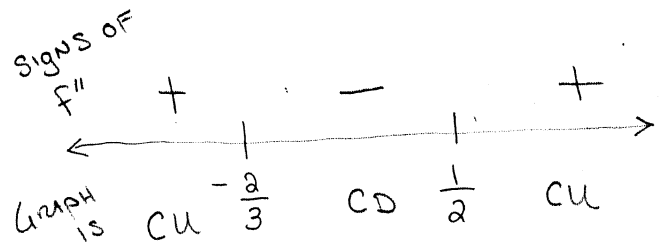
$f(\frac{-1+\sqrt{65}}{8}) \approx -1.3318$  IS A REL MIN

$$f''(x) = 72x^2 + 12x - 24$$

$$= 12(6x^2 + x - 2)$$

$$= 12(3x+2)(2x-1) = 0$$

$$x = -\frac{2}{3}, \quad x = \frac{1}{2}$$



GRAPH IS CONCAVE UP ON  $(-\infty, -\frac{2}{3}) \cup (\frac{1}{2}, \infty)$ .

GRAPH IS CONCAVE DOWN ON  $(-\frac{2}{3}, \frac{1}{2})$ .

$(-\frac{2}{3}, -\frac{47}{27})$  &  $(\frac{1}{2}, \frac{5}{8})$

ARE INFLECTION POINTS.

TURN OVER.

2. (2 points) The table below gives the values of a function  $f$  and its first two derivatives at selected values of  $x$ . Determine which row gives the data for  $f$ , which row gives the data for  $f'$ , and which row gives the data for  $f''$ . Explain your reasoning.

$x$	0.00	0.33	0.66	1.00	1.33	1.66	2.00	2.33	2.66	3.00
$A(x)$	0.00	0.64	1.14	1.38	1.28	0.84	0.08	-0.89	-1.91	-2.83
$B(x)$	0.00	0.11	0.41	0.84	1.30	1.66	1.82	1.69	1.22	0.42
$C(x)$	2.00	1.78	1.16	0.24	-0.83	-1.85	-2.65	-3.07	-3.00	-2.40

$$B(x) = f(x)$$

$$A(x) = f'(x)$$

$$C(x) = f''(x)$$

When  $B$  is increasing (on  $(0, 2)$ ),

$A$  is positive. When  $B$  is decreasing (on  $(2, 3)$ ),

$A$  is negative.

This supports  $B'(x) = A(x)$ .

When  $A$  is decreasing (on  $(0, 1)$ ),

$C$  is positive. When  $A$  is increasing (on  $(1, 3)$ ),

$C$  is negative.

This supports  $A'(x) = C(x)$ .