Math 171 - Test 1

September 19, 2018

Name key

Score ______

Show all work to receive full credit. Supply explanations where necessary. When evaluating limits, you may need to use $+\infty$, $-\infty$, or DNE (does not exist).

1. (6 points) Use a table of numerical values to approximate the following limit. Your table must show function values at six or more points.

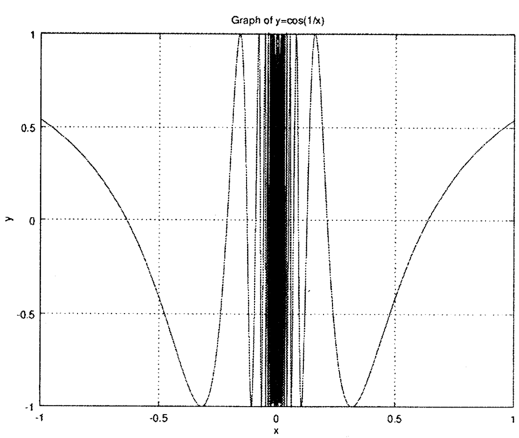
$$\lim_{x\to 2} \underbrace{\frac{\frac{x}{x+1}-\frac{2}{3}}{x-2}}_{f(x)}$$

X	f(x)
1.9	0.11494
1.99	0.11148
1.999	0.11115

IT LOOKS LIKE $\lim_{x\to 2} f(x) \approx 0.111$

X	f(x)
2.1	0.10753
2.01	0.11074
2.001	0.11107

2. (6 points) A rough sketch of the graph of $y = \cos(1/x)$ is shown below.



(a) Carefully explain why $\lim_{x\to 0} \cos(1/x)$ does not exist.

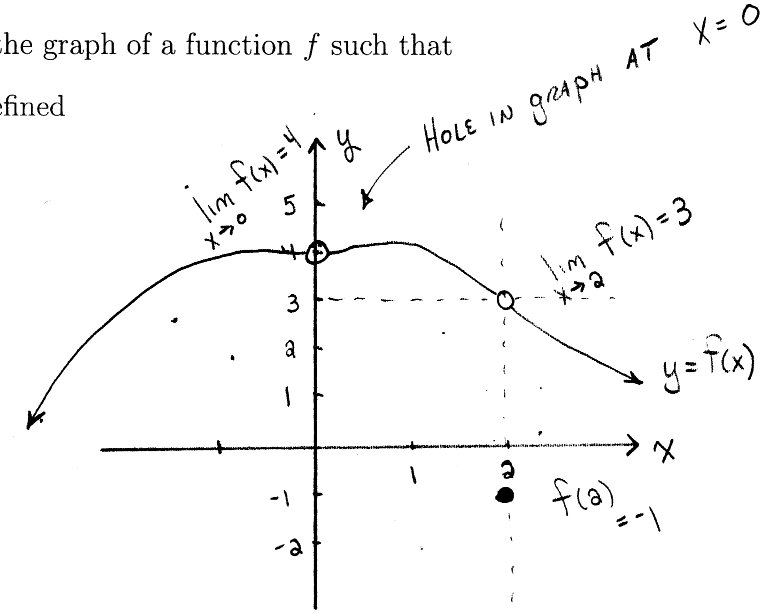
AS X APPROACHES 0, THE VALUES OF $\cos(1/x)$ OSCILLATE WILDLY BETWEEN -1 AND 1. THE CLOSER X GETS TO 0, THE MORE OSCILLATIONS OCCUR. (THIS WAS OUR REASON #3.)

(b) Determine the limit: $\lim_{x\to 0.1} \cos(1/x)$

$$\lim_{x\to 0.1} \cos(1/x) = \cos(10) \approx -0.83907$$

3. (6 points) Sketch the graph of a function f such that

- $f(0)$ is not defined
- $\lim_{x\to 0} f(x) = 4$
- $f(2) = -1$
- $\lim_{x\to 2} f(x) = 3$



4. (6 points) Explain why $\lim_{x\to 2} \dfrac{2x^2-4x}{|x-2|}$ fails to exist. Show work to justify your answer.

$$\lim_{x\to 2^+} \frac{2x^2-4x}{|x-2|} = \lim_{x\to 2} \frac{2x\cancel{(x-2)}}{\cancel{x-2}} = 4$$

$$\lim_{x\to 2^-} \frac{2x^2-4x}{|x-2|} = \lim_{x\to 2} \frac{2x\cancel{(x-2)}}{-\cancel{(x-2)}} = -4$$

LIMIT FROM LEFT $\neq$ LIMIT FROM RIGHT

5. (6 points) Is g continuous at $x = 4$? Show work and explain your reasoning. (Your work should address the three things we check when assessing continuity at a point.)

$$g(x) = \begin{cases} x^2 - 8x + 15, & x \le 4 \\ 3 - x\cos \pi x, & x > 4 \end{cases}$$

1. $g(4) = 4^2 - 8(4) + 15$
   $= -1$

   $g(4) = -1$

2. $\lim_{x\to 4^-} g(x) = \lim_{x\to 4} (x^2 - 8x + 15)$
   $= 16 - 32 + 15 = -1$

   $\lim_{x\to 4^+} g(x) = \lim_{x\to 4} 3 - x\cos \pi x$
   $= 3 - 4\cos 4\pi$
   $= 3 - 4 = -1$

   $\lim_{x\to 4} g(x) = -1$

3. $\lim_{x\to 4} g(x) = g(4)$

6. (24 points) Determine each limit analytically, or explain why the limit does not exist. You may need to use $+\infty$, $-\infty$, or DNE.

(a) $\lim_{x \to 0} \frac{\tan \pi x}{3x}$ $\quad \frac{0}{0}$

$$= \lim_{x \to 0} \frac{\sin \pi x}{3x \cos \pi x} = \lim_{x \to 0} \frac{\pi}{3 \cos \pi x} \frac{\sin \pi x}{\pi x}$$

$\cos \pi x \to 1$, $\frac{\sin \pi x}{\pi x} \to 1$

$$= \boxed{\frac{\pi}{3}}$$

(b) $\lim_{y \to 5^+} \left(\frac{2y - 10}{y^2 - 10y + 25} \right)$ $\quad \frac{0}{0}$

$$= \lim_{y \to 5^+} \frac{2(y-5)}{(y-5)^2} = \lim_{y \to 5^+} \frac{2}{y-5}$$

$\frac{2}{0}$ $\quad \frac{+}{+} = +$

$$= \boxed{+\infty}$$

(c) $\lim_{r \to 0^+} \frac{\sqrt{2+r} - \sqrt{2}}{r}$ $\quad \frac{0}{0}$

$$\cdot \frac{\sqrt{2+r} + \sqrt{2}}{\sqrt{2+r} + \sqrt{2}} = \lim_{r \to 0^+} \frac{2 + r - 2}{r(\sqrt{2+r} + \sqrt{2})}$$

$$= \lim_{r \to 0^+} \frac{\cancel{r}}{\cancel{r}(\sqrt{2+r} + \sqrt{2})} = \boxed{\frac{1}{2\sqrt{2}}}$$

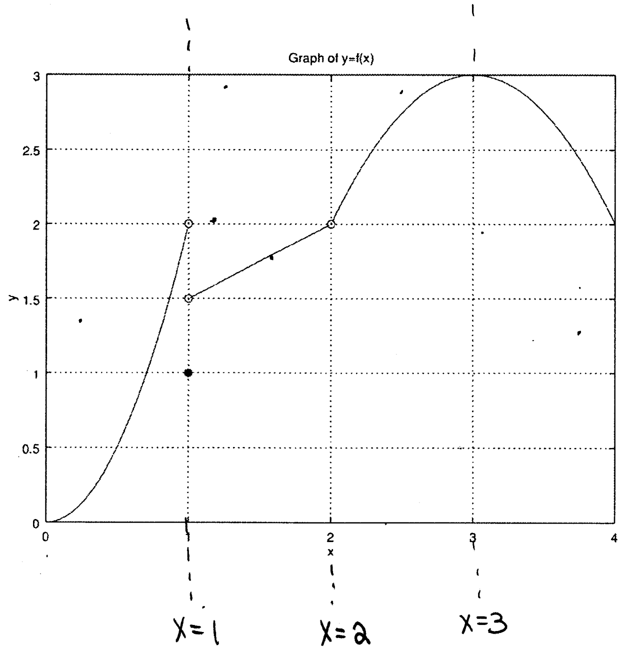
(d) $\lim_{x \to -3} \frac{(x+2)(x+7)+4}{x+3}$ $\quad \frac{0}{0}$

$$= \lim_{x \to -3} \frac{x^2 + 9x + 14 + 4}{x+3} = \lim_{x \to -3} \frac{x^2 + 9x + 18}{x+3} = \lim_{x \to -3} \frac{\cancel{(x+3)}(x+6)}{\cancel{x+3}}$$

$$= -3 + 6$$

$$= \boxed{3}$$

7. (10 points) Referring to the graph of $y = f(x)$ shown below, determine each of the following or explain why it does not exist.



(a) $\lim_{x \to 1} f(x)$.

DNE $\quad \lim_{x \to 1^-} f(x) = 2 \neq \lim_{x \to 1^+} f(x) = 1.5$

(b) $f(1)$ $= 1$

(c) $\lim_{x \to 3^+} f(x) = 3$

(d) $\lim_{x \to 2} f(x) = 2$

(e) $f(2)$ DNE THERE IS A HOLE AT $X = 2$ --- NO FUNCTION VALUE!

2 DISCONTS
$X=1, X=2$

8. (2 points) Refer to the function $y = f(x)$ whose graph is shown above. Classify the discontinuities of f.

NONREMOVABLE, jump DISCONT AT $X = 1$

REMOVABLE DISCONT AT $X = 2$

9. (8 points) Determine all vertical asymptotes of the graph of $h(x) = \dfrac{x^2 + 2x - 8}{x^2 - 4}$.

$$h(x) = \frac{(x+4)(x-2)}{(x+2)(x-2)} = \frac{x+4}{x+2}, \quad x \neq \pm 2$$

REMOVABLE DISCONT AT $x = 2$.

THERE IS A SINGLE VERTICAL ASYMPTOTE: $x = -2$

Follow-up question: How do you know that each answer above is indeed a vertical asymptote?

$\frac{2}{0}$ $\frac{+}{+} = +$

$$\lim_{x \to -2^+} \frac{x+4}{x+2} = +\infty$$

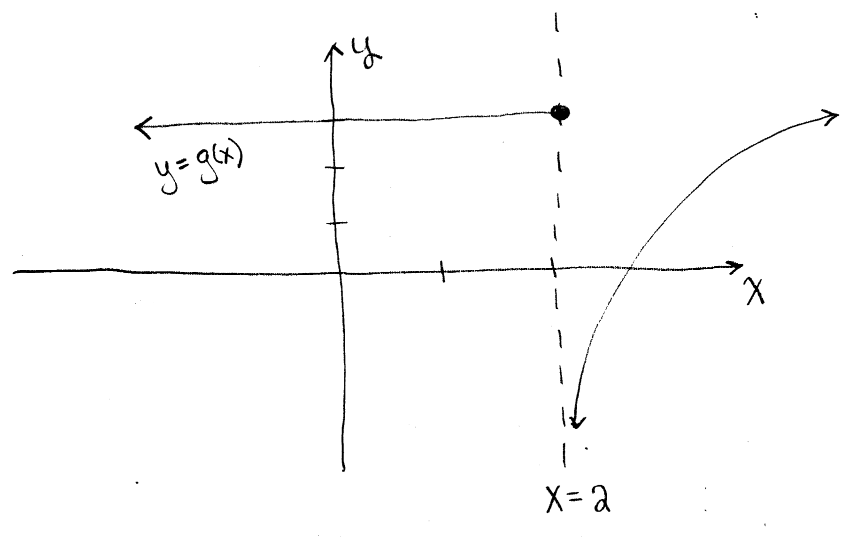
$\frac{2}{0}$ $\frac{+}{-} = -$

$$\lim_{x \to -2^-} \frac{x+4}{x+2} = -\infty$$

INFINITE ONE-SIDED LIMITS $\Rightarrow$ $x = -2$ IS A VA

10. (6 points) Sketch the graph of a function g that satisfies both of the following conditions:

- $\lim_{x \to 2^+} g(x) = -\infty$
- $g(2) = 3$



Follow-up question: Refer to your function (and graph) above. Is the line $x = 2$ a vertical asymptote of the graph of g? Briefly explain.

YES, IT IS. X-VALUES AT WHICH A FUNCTION HAS A ONE- OR TWO-SIDED INFINITE LIMIT GIVE RISE TO VA'S.

11. (6 points) Consider the function $f(x) = 5 + 3x\sin\pi x,\ x \geq 2.$ Determine each of the following or explain why it does not exist.

(a) $f(1)$ DNE

BECAUSE f IS ONLY DEFINED WHEN $X \geq 2$.

(b) $\lim_{x\to 2^+} f(x)$ $= \lim_{X\to 2} (5 + 3x \sin \pi X)$

$= 5 + 6 \sin 2\pi = \boxed{5}$

(c) $\lim_{x\to 2} f(x)$ DNE BECAUSE

f IS NOT DEFINED ON BOTH SIDES OF $X=2$, AND THIS IS ASKING FOR A TWO-SIDED LIMIT.

12. (6 points) Polynomial functions are continuous at all real numbers. Give an example of a polynomial function, and explain how you know it is continuous at any value of x.

$f(x) = 3x^3 - 2x^2 + x - 1.$

Suppose I PICK ANY NUMBER, SAY C.

I CAN GET THE LIMIT AT $X = C$ BY SUBSTITUTING

$\lim_{X\to C} f(x) = 3c^3 - 2c^2 + c - 1 = f(c)$

THIS IS EXACTLY WHAT IT MEANS TO BE CONT. AT C.

13. (2 points) The function $y = \dfrac{\sin x}{x}$ is discontinuous at $x = 0$. What kind of discontinuity do you find at $x = 0$, and how do you know?

IT IS A REMOVABLE DISCONT. BECAUSE THE LIMIT EXISTS AT $X = 0$:

$$\lim_{X\to 0} \frac{\sin X}{X} = 1.$$

14. (6 points) Determine whether each statement is true (T) or false (F).

(a) F If f is defined at $x = c$, then $\lim_{x \to c} f(x)$ exists.

(b) F $\lim_{x \to 0} \sqrt{x} = 0$ $\lim_{x \to 0^-} \sqrt{x}$ DNE!

SAME IDEAS! LIMIT HAS NOTHING TO DO WITH FUNC. VALUE!

(c) F If $h(-2) = 0$, then $\lim_{x \to -2} h(x) = 0$.

(d) T If $\lim_{x \to 1} f(x) = f(1)$, then f is continuous at $x = 1$.   DEF. OF CONTINUITY.

(e) T If $\lim_{t \to 0} h(t) = 5$, then $\lim_{t \to 0^-} h(t) = 5$.   IF 2-SIDED LIMIT IS L, BOTH 1-SIDED LIMITS ARE L.

(f) F If f and g are polynomials and $g(3) = 0$, then the graph of $\dfrac{f(x)}{g(x)}$ must have a vertical asymptote at $x = 3$.

NOT NECESSARILY, BUT DEFINITELY IF $f(3) \neq 0$.