Math 171 - Final Exam

December 10, 2018

Name _____

Score _____

Show all work to receive full credit. Supply explanations where necessary.

1. (10 points) The graph of $y = f(x)$ is shown below, with dashed lines showing asymptotes. Use the graph to estimate each limit. Use $+\infty$, $-\infty$, or DNE if appropriate.

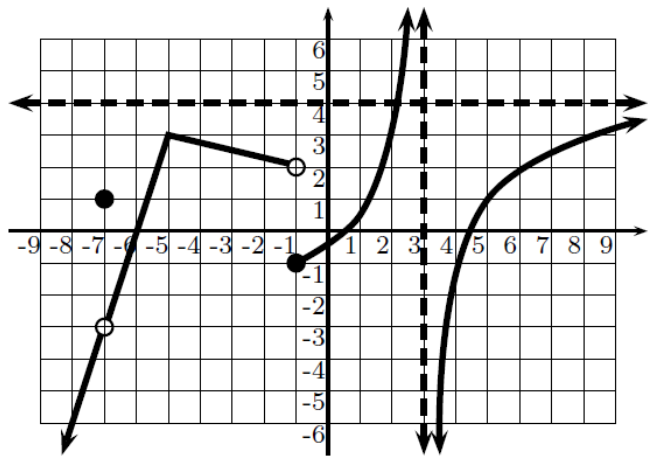
   (a) $\lim_{x \to -7} f(x)$

   (b) $\lim_{x \to -1} f(x)$

   (c) $\lim_{x \to 5} f(x)$

   (d) $\lim_{x \to 3^+} f(x)$

   (e) $\lim_{x \to \infty} f(x)$



2. (6 points) Refer to the graph of $y = f(x)$ shown above.

   (a) State each x-value at which f is discontinuous. For each discontinuity, tell whether it is removable or nonremovable.

   (b) State each x-value at which f is not differentiable.

3. (6 points) Evaluate the limit analytically: $\lim_{x \to 3} \dfrac{x(x-1)-6}{x^2-7x+12}$.

4. (10 points) Differentiate each function. Do not simplify.

   (a) $f(x) = 8x^3(5x-2)^4$

   (b) $g(x) = \dfrac{7\cos 3x}{\sqrt{x}}$

5. (8 points) Find an equation of the line tangent to the graph of $y = \frac{2}{\pi}x^2 + \tan x$ at the point where $x = \pi$.

6. (10 points) Assume that y is implicitly defined as a function of x by the equation $7x + x^2y = 4 + 3y^2$. Find dy/dx.

7. (18 points) Let $f(x) = x^3 - 9x^2 + 15x + 3$. Find open intervals on f is increasing/decreasing and determine the relative extreme values. Then find open intervals on which the graph of f is concave up/down and locate any points of inflection.

8. (12 points) Find the area of the bounded region between the graphs of $y = x^2 - 2$ and $y = x$.

9. (8 points) Evaluate the indefinite integral: $\displaystyle\int 6x(x^2+9)^3\,dx$.

10. (8 points) Use the **limit definition of derivative** to determine $f'(x)$ if $f(x) = x^2 + x$. (You will receive points only if you use the limit definition.)

11. (12 points)

(a) Use four subintervals to compute a Riemann sum that approximates $\int_1^2 (x-1)\,dx$.

(b) **Use area** to determine the exact value of $\int_1^2 (x-1)\,dx$.

The remaining problems all have multiple-choice solutions. Each problem is worth 3 points. **You must show your work to receive full credit.** The answer itself is worth 1 point. Your work/explanation is worth 2 points.

1. Use Newton's method staring with $x_0 = 2$ to approximate a solution of $x^2 = \cos x$. Which of these is closest to your value of x_2?

   (a) 1.10045

   (b) 0.82413

   (c) 0.85539

   (d) 0.73911

2. Which one of the following is $\lim_{x \to 5^+} \frac{7x}{x-5}$? (Show work or explain.)

   (a) ∞

   (b) $-\infty$

   (c) 0

   (d) 7

3. The height of an object launched upward is given by $h(t) = -16t^2 + 64t + 80$, where $h(t)$ represents height (in feet) at time t (in seconds). What is the maximum height of the object?

   (a) 80 feet

   (b) 144 feet

   (c) 64 feet

   (d) 145 feet

4. Which of the following equations best relates the rates of change with respect to time t of the area A and the radius r of a circle?

   (a) $A = \pi r^2$

   (b) $\frac{dA}{dt} = \pi \frac{dr}{dt}$

   (c) $\frac{dA}{dt} = 2\pi \frac{dr}{dt}$

   (d) $\frac{dA}{dt} = 2\pi r \frac{dr}{dt}$

5. If f is a function with $f'(4) = 0$ and $f''(4) = 6$, then which one of the following must be true?

   (a) $f(4)$ is a relative maximum.
   (b) $f(4)$ is a relative minimum.
   (c) The graph of f has an inflection point at $x = 4$.
   (d) f is increasing at $x = 4$.

6. Which one of the following is $\lim_{x\to\infty} \dfrac{-10x+3}{5x^2+x+6}$? (Show work or explain.)

   (a) ∞
   (b) $-\infty$
   (c) 0
   (d) -2

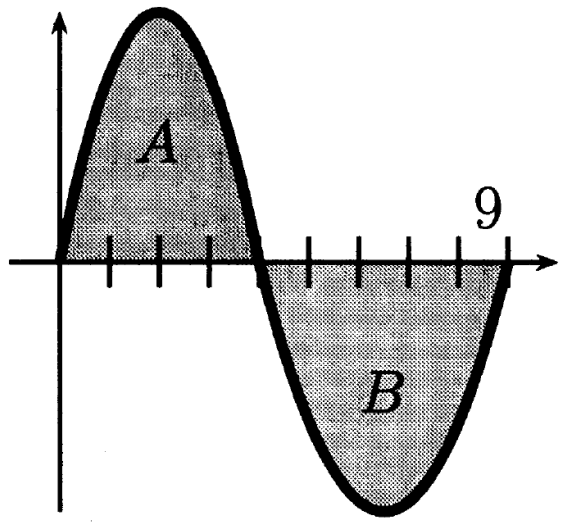
7. Suppose f is a function with $f(1) = 2$ and $\lim_{x\to 1^-} f(x) = -\infty$. Which one of the following must be true?

   (a) $x = 1$ is a vertical asymptote of the graph of f.
   (b) $y = 2$ is a horizontal asymptote of the graph of f.
   (c) $\lim_{x\to 1^+} f(x) = +\infty$
   (d) f is differentiable at $x = 1$.

8. Which one of the following best describes the line tangent to the graph of $f(x) = (3x-12)^{1/3}$ at the point $(4, 0)$?

   (a) The tangent line is horizontal.
   (b) The tangent line is vertical.
   (c) A unique tangent line does not exist.
   (d) The tangent line cannot be determined from the given information.

9. The graph of $y = f(x)$ is shown below. Region A has area 7, and region B has area 8. Which one of the following is the value of $\int_0^9 f(x)\,dx$?



(a) 15
(b) 1
(c) -1
(d) 0

10. Use calculus techniques to find the absolute maximum value of $f(x) = 2x^3+3x^2-12x+6$ on the interval $[-4, 2]$.

(a) -26
(b) -2
(c) 26
(d) 19

11. Use the Fundamental Theorem of Calculus to evaluate $\int_0^\pi \sin x\,dx$.

(a) 1
(b) 0
(c) 1.975
(d) 2

12. Let $r(x) = x^3 + \sin(10x)$. Without looking at the graph of r, determine whether the graph is concave up or down at the point where $x = 0.63$.

(a) concave down

(b) concave up

(c) There is an inflection point at 0.63.

(d) r is not defined at $x = 0.63$.

13. Use differentials to approximate the change in $f(x) = 2x^3 - \sqrt[3]{x}$ as x changes from $x = 1$ to $x = 1.1$.

(a) $\Delta y \approx 0.62972$

(b) $\Delta y \approx 1.62972$

(c) $\Delta y \approx 0.69472$

(d) $\Delta y \approx 0.56667$

14. Suppose you used a table of numerical values to approximate $\lim_{x \to 0} \frac{2^x - 1}{x}$. Which one is these is the most reasonable estimate for the limit?

(a) 0.70000

(b) 0.69555

(c) 0.69315

(d) The limit does not exist.