

Math 172 - Quiz 10

November 9, 2016

Name key

Score _____

Show all work to receive full credit. Supply explanations when necessary.

1. (4 points) Evaluate $\int_0^3 \frac{x^3}{\sqrt{x^2+9}} dx$.

$$x = 3 \tan \theta, \quad -\frac{\pi}{2} < \theta < \frac{\pi}{2}$$

$$dx = 3 \sec^2 \theta d\theta$$

$$x = 0 \Rightarrow \theta = 0$$

$$x = 3 \Rightarrow \tan \theta = 1 \\ \Rightarrow \theta = \frac{\pi}{4}$$

$$\int_0^{\pi/4} \frac{27 \tan^3 \theta (3 \sec^2 \theta)}{\sqrt{9 \sec^2 \theta}} d\theta$$

$$\int_0^{\pi/4} \frac{27 \tan^3 \theta (3 \sec^2 \theta)}{|3 \sec \theta|} d\theta$$

$$\sec \theta > 0 \\ \text{on } [0, \frac{\pi}{4}]$$

$$27 \int_0^{\pi/4} \tan^3 \theta \sec \theta d\theta \\ = 27 \int_0^{\pi/4} (\sec^2 \theta - 1) \tan \theta \sec \theta d\theta$$

$$u = \sec \theta$$

$$du = \sec \theta \tan \theta d\theta$$

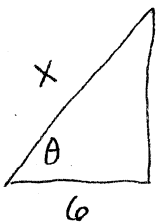
$$\theta = 0 \Rightarrow u = 1$$

$$\theta = \frac{\pi}{4} \Rightarrow u = \sqrt{2}$$

$$27 \int_1^{\sqrt{2}} (u^2 - 1) du$$

$$= 27 \left(\frac{u^3}{3} - u \right) \Big|_1^{\sqrt{2}} \\ \approx 5.27$$

2. (1 point) After making the trigonometric substitution $x = 6 \sec \theta$, you evaluated an integral and obtained $\theta + \cot \theta + C$. Resubstitute and write your result in terms of the variable x .



$$\sqrt{x^2 - 36}$$

Assuming

$$x > 0$$

$$\theta + \cot \theta + C$$

$$= \sec^{-1} \frac{x}{6} + \frac{6}{\sqrt{x^2 - 36}} + C$$

3. (4 points) Evaluate $\int \frac{3x^2 - 8x + 2}{x^2 - 3x} dx$.

$$\begin{array}{r} 3 \\ x^2 - 3x \overline{) 3x^2 - 8x + 2} \\ \underline{-(3x^2 - 9x)} \\ x + 2 \end{array}$$

$$3 + \frac{x+2}{x^2-3x} = 3 + \frac{x+2}{x(x-3)}$$

$$\frac{x+2}{x(x-3)} = \frac{A}{x} + \frac{B}{x-3}$$

$$x+2 = A(x-3) + Bx$$

$$x=0: 2 = -3A \Rightarrow A = -\frac{2}{3}$$

$$x=3: 5 = 3B \Rightarrow B = \frac{5}{3}$$

$$\int \left(3 - \frac{2/3}{x} + \frac{5/3}{x-3} \right) dx$$

$$= 3x - \frac{2}{3} \ln|x| + \frac{5}{3} \ln|x-3| + C$$

4. (1 point) Write the form of the partial fraction decomposition of $\frac{x}{x^3(x^2+9)^2(2x+1)}$.

Do not solve for the undetermined coefficients.

$$\frac{A}{x} + \frac{B}{x^2} + \frac{C}{x^3} + \frac{Dx+E}{x^2+9} + \frac{Fx+G}{(x^2+9)^2} + \frac{H}{2x+1}$$