

Math 172 - Quiz 11

November 16, 2016

Name key

Score _____

Show all work to receive full credit. Supply explanations when necessary.

1. (2 points) Evaluate the limit: $\lim_{x \rightarrow 0} \frac{5x}{\tan^{-1} 2x}$ $\frac{0}{0}$

$$\begin{aligned} &= \lim_{x \rightarrow 0} \frac{\frac{5}{2}}{\frac{1}{1+4x^2}} = \lim_{x \rightarrow 0} \frac{5(1+4x^2)}{2} \\ &= \boxed{\frac{5}{2}} \end{aligned}$$

2. (2 points) Evaluate the limit: $\lim_{x \rightarrow 0^+} \frac{e^x - (1+x)}{x^3}$ $\frac{0}{0}$

$$\begin{aligned} &= \lim_{x \rightarrow 0^+} \frac{e^x - 1}{3x^2} \quad \frac{0}{0} \\ &= \lim_{x \rightarrow 0^+} \frac{e^x}{6x} = \boxed{+\infty} \end{aligned}$$

3. (3 points) Evaluate the limit:

$$\lim_{x \rightarrow \infty} \frac{1 - \exp\left(-\frac{1}{2x} - \frac{1}{2xe-2}\right)}{\frac{1}{xe}} \quad \text{o/o}$$

$$\lim_{x \rightarrow \infty} \frac{-\exp\left(-\frac{1}{2x} - \frac{1}{2xe-2}\right) \left[\frac{1}{2x^2} + \frac{2e}{(2xe-2)^2} \right]}{\frac{1}{x^2 e}}$$

$$= \lim_{x \rightarrow \infty} \cancel{\exp\left(-\frac{1}{2x} - \frac{1}{2xe-2}\right)} \cdot \lim_{x \rightarrow \infty} \frac{\frac{1}{2x^2} + \frac{2e}{(2xe-2)^2}}{\frac{1}{x^2 e}} \quad \text{o/o}$$

$$= \lim_{x \rightarrow \infty} \frac{x^2 e}{2x^2} + \frac{2x^2 e^2}{(2xe-2)^2}$$

$$= \frac{e}{2} + \frac{2e^2}{4e^2} = \boxed{\frac{e}{2} + \frac{1}{2}}$$

4. (3 points) Evaluate the limit:

$$\lim_{x \rightarrow \infty} x \tan \frac{1}{x}$$

Form $\infty \cdot 0$

$$\lim_{x \rightarrow \infty} \frac{\tan \frac{1}{x}}{\frac{1}{x}}$$

Let $u = \frac{1}{x}$

As $x \rightarrow \infty$, $u \rightarrow 0^+$

$$= \lim_{u \rightarrow 0^+} \frac{\tan u}{u} \quad \text{o/o}$$

$$= \lim_{u \rightarrow 0^+} \frac{\sec^2 u}{1} = \boxed{1}$$