Math 172 - Quiz 12

November 30, 2016

Name key

Score ________

Show all work to receive full credit. Supply explanations when necessary.

1. (3 points) Pick a positive number k. (For real! Pick an actual number.) Consider the sequence $\{a_n\}_{n=1}^{\infty}$ with $a_1 = \sqrt{k}$ and $a_{n+1} = \sqrt{k + a_n}$. Your sequence is increasing and bounded above (you don't have to prove it). Find the limit of your sequence.

I'LL JUST STICK WITH k.

$$\underbrace{\lim_{n\to\infty} a_{n+1}} = \sqrt{k + \lim_{n\to\infty} a_n}$$

$$L = \sqrt{k + L}$$

OR

$$L^2 - L - k = 0$$

$$\boxed{L = \frac{1 + \sqrt{1 + 4k}}{2}}$$

FOR EXAMPLE, IF $k = 2$,

$$L = \frac{1 + \sqrt{9}}{2} = 2$$

OR IF $k = 6$,

$$L = \frac{1 + \sqrt{25}}{2} = 3$$

2. (2 points) Find the sum of the series $\displaystyle\sum_{n=3}^{\infty} \frac{8}{3^n}$.

$$\sum_{n=0}^{\infty} \frac{8}{3^n} = 8 + \frac{8}{3} + \frac{8}{9} + \sum_{n=3}^{\infty} \frac{8}{3^n}$$

$$\frac{8}{1 - \frac{1}{3}} = 8 + \frac{8}{3} + \frac{8}{9} + \sum_{n=3}^{\infty} \frac{8}{3^n}$$

$$\sum_{n=3}^{\infty} \frac{8}{3^n} = 12 - 8 - \frac{8}{3} - \frac{8}{9} = \boxed{\frac{4}{9}}$$

3. (2 points) Find the sum of the series $\sum_{n=1}^{\infty} \frac{1}{(2n+1)(2n+3)}$.

$$\frac{1}{(2n+1)(2n+3)} = \frac{A}{2n+1} + \frac{B}{2n+3}.$$

$$1 = A(2n+3) + B(2n+1)$$

$$n = -\frac{3}{2}: \quad 1 = -2B \Rightarrow B = -\frac{1}{2}$$

$$n = -\frac{1}{2}: \quad 1 = 2A \Rightarrow A = \frac{1}{2}$$

$$\sum_{n=1}^{\infty} \left(\frac{1}{2} \frac{1}{2n+1} - \frac{1}{2} \frac{1}{2n+3} \right) = \left(\frac{1}{6} - \frac{1}{10}\right) + \left(\frac{1}{10} - \frac{1}{14}\right) + \left(\frac{1}{14} - \frac{1}{18}\right) + \cdots$$

$$= \frac{1}{6}$$

4. (3 points) Use the integral test to determine whether the series converges or diverges: $\sum_{n=2}^{\infty} \frac{2}{n\sqrt{\ln n}}$.

$$f(x) = \frac{2}{x(\ln x)^{1/2}}$$

POS, DECREASING & CONT FOR $X > 1$

$$\int_2^{\infty} \frac{2}{x(\ln x)^{1/2}}\,dx = \lim_{c\to\infty} \int_2^{c} \frac{2}{x(\ln x)^{1/2}}\,dx$$

$u = \ln x$

$du = \frac{1}{x}\,dx$

$$\lim_{c\to\infty} \int_{\ln 2}^{\ln c} 2u^{-1/2}\,du = \lim_{c\to\infty} 4u^{1/2} \Big|_{\ln 2}^{\ln c}$$

$$= \lim_{c\to\infty} 4(\ln c)^{1/2} - 4(\ln 2)^{1/2}$$

$$= +\infty$$

SERIES DIVERGES.