

Math 172 - Quiz 1

August 24, 2016

Name key

Score _____

Show all work to receive full credit. Supply explanations when necessary. YOU MUST WORK INDIVIDUALLY.

1. (3 points) Find an equation of the line tangent to the graph of $y = \ln\left(\frac{x^2}{x+3}\right)$ at the point where $x = 1$.

$$y = 2 \ln x - \ln(x+3)$$

When $x=1$, $y = \ln\left(\frac{1}{4}\right) = -\ln 4$

$$\frac{dy}{dx} = \frac{2}{x} - \frac{1}{x+3}$$

TANGENT LINE:

$$y + \ln 4 = \frac{7}{4}(x-1)$$

$$m = \left. \frac{dy}{dx} \right|_{x=1} = \frac{2}{1} - \frac{1}{4} = \frac{7}{4}$$

or

$$y = \frac{7}{4}x - \frac{7}{4} - \ln 4$$

2. (3 points) Use logarithmic differentiation to compute $f'(3)$.

$$y = \frac{x(x-1)^3}{(x+1)^{1/2}}$$

$$f(x) = \frac{x(x-1)^3}{\sqrt{x+1}}$$

$$\ln y = \ln x + 3 \ln(x-1) - \frac{1}{2} \ln(x+1)$$

$$\frac{1}{y} \frac{dy}{dx} = \frac{1}{x} + \frac{3}{x-1} - \frac{1/2}{x+1}$$

$$\frac{dy}{dx} = y \left(\frac{1}{x} + \frac{3}{x-1} - \frac{1/2}{x+1} \right)$$

$$f'(3) = f(3) \left(\frac{1}{3} + \frac{3}{2} - \frac{1/2}{4} \right)$$

$$= \frac{3(8)}{2} \left(\frac{41}{24} \right) = \frac{41}{2}$$

3. (4 points) Find the area of the region under the graph of $y = \frac{1}{x \ln x^3}$ over the interval from $x = 2$ to $x = 3$.

$$\int_2^3 \frac{1}{x \ln x^3} dx = \frac{1}{3} \int_2^3 \frac{1}{x \ln x} dx$$

$$u = \ln x$$

$$du = \frac{1}{x} dx$$

$$= \frac{1}{3} \int_{\ln 2}^{\ln 3} \frac{1}{u} du = \frac{1}{3} \ln |u| \Big|_{\ln 2}^{\ln 3}$$

$$= \frac{1}{3} [\ln(\ln 3) - \ln(\ln 2)]$$

$$\approx 0.1535$$