Math 172 - Quiz 2

August 31, 2016

Name key

Score _____

Show all work to receive full credit. Supply explanations when necessary.

1. (3 points) Evaluate the definite integral: $\int_1^e \frac{(1+\ln x)^2}{x}\,dx$

$u = 1 + \ln x$

$du = \frac{1}{x}\,dx$

$\longrightarrow \int_{u=1}^{u=2} u^2\,du = \frac{1}{3}u^3\Big|_1^2 = \frac{8}{3} - \frac{1}{3} = \boxed{\frac{7}{3}}$

2. (3 points) Explain how you know that $f(x) = 3 - x - x^5$ has an inverse. Without finding the inverse function, compute $f^{-1}(3)$.

$f'(x) = -1 - 5x^4 < 0$

f' IS NEGATIVE $\Rightarrow$ f IS DECREASING $\Rightarrow$ f IS 1-1 $\Rightarrow$ f HAS AN INVERSE

$f^{-1}(3) = y$

$\Updownarrow$

$f(y) = 3$

$3 - y - y^5 = 3$

$\Updownarrow$

$y = 0$ $\quad \boxed{f^{-1}(3) = 0}$

3. (4 points) Let $g(x) = x^3 + 2x - 1$. Compute $\frac{d}{dx}g^{-1}(x)\Big|_{x=2}$.

$\frac{1}{g'(g^{-1}(2))} = \frac{1}{3(1)^2 + 2} = \boxed{\frac{1}{5}}$

$g^{-1}(2) = y$

$\Updownarrow$

$g(y) = 2$

$y^3 + 2y - 1 = 2 \Rightarrow y = 1$

$g'(x) = 3x^2 + 2$