Math 172 - Quiz 3

Name key

September 7, 2016

Score _____

Show all work to receive full credit. Supply explanations when necessary. YOU MUST WORK INDIVIDUALLY.

1. (3 points) Find an equation of the line tangent to the graph of $y = (\sin x)^{2x}$ at the point where $x = \pi/2$. (Hint: Use logarithmic differentiation.)

$\ln y = 2x \ln \sin x$

$\frac{1}{y}\frac{dy}{dx} = 2\ln \sin x + \frac{2x\cos x}{\sin x}$

$\frac{dy}{dx} = y\left(2\ln \sin x + \frac{2x\cos x}{\sin x}\right)$

When $x = \frac{\pi}{2}$, $y = 1$ & $\frac{dy}{dx} = 1(2(0)+0) = 0$

TANGENT LINE IS $y = 1$

2. (4 points) Find the relative extreme values of the function $g(x) = -2 + e^{3x}(4 - 2x)$.

$g'(x) = 3e^{3x}(4-2x) - 2e^{3x}$

$= 10e^{3x} - 6xe^{3x}$

$= 2e^{3x}(5-3x) = 0$

$\Rightarrow x = \frac{5}{3}$

SIGNS OF $g'(x)$: + | − (at $x = 5/3$)

g INC $5/3$ g DEC

$g\left(\frac{5}{3}\right) = -2 + \frac{2}{3}e^5 \approx 96.9$

IS A REL MAX

3. (3 points) Evaluate the definite integral: $\int_{1/2}^{1} \frac{5e^{1/x^2}}{x^3}\,dx$

$u = \frac{1}{x^2}$

$du = \frac{-2}{x^3}dx$

$-\frac{5}{2}du = \frac{5}{x^3}dx$

$x = \frac{1}{2} \Rightarrow u = 4$

$x = 1 \Rightarrow u = 1$

$= -\frac{5}{2}\int_4^1 e^u\,du$

$= -\frac{5}{2}e^u\Big|_4^1$

$= -\frac{5}{2}(e - e^4)$

≈ 129.7