Math 172 - Quiz 4

September 14, 2016

Name __key__

Score _____

Show all work to receive full credit. Supply explanations when necessary.

1. (3 points) Find an equation of the line tangent to the graph of the equation

$$\tan^{-1}(xy) = \sin^{-1}(x+y)$$

at the point $(0,0)$. (Hint: Use implicit differentiation.)

$$\frac{1}{1+(xy)^2}\left(y + x\frac{dy}{dx}\right) = \frac{1}{\sqrt{1-(x+y)^2}}\left(1 + \frac{dy}{dx}\right)$$

AT $(0,0)$ ...

$$\frac{1}{1+0^2}(0+0) = \frac{1}{\sqrt{1-0^2}}\left(1 + \frac{dy}{dx}\right)$$

$$\Downarrow$$

$$\frac{dy}{dx} = -1$$

TANGENT LINE IS

$$y - 0 = -1(x-0)$$

OR

$$\boxed{y = -x}$$

2. (2 points) Find the slope of the line tangent to the graph of $y = 4x\cos^{-1}(x-1)$ at the point where $x = 1$.

$$\frac{dy}{dx} = 4\cos^{-1}(x-1) + \frac{-4x}{\sqrt{1-(x-1)^2}}$$

$$\frac{dy}{dx}\bigg|_{x=1} = 4\cos^{-1}(0) + -4$$

$$= 4\left(\frac{\pi}{2}\right) - 4 = \boxed{2\pi - 4}$$

3. (3 points) Evaluate the indefinite integral: $\displaystyle\int \frac{2x-5}{x^2+2x+2}\,dx$

$\underbrace{x^2+2x+2}_{(x+1)^2+1}$

$u = x+1 \quad \Rightarrow \quad 2x-5 = 2u-7$

$du = dx$

$\displaystyle\int \frac{2u-7}{u^2+1}\,du$

$= \displaystyle\int \frac{2u}{u^2+1}\,du - 7\int \frac{1}{u^2+1}\,du$

$\underbrace{\phantom{\int \frac{2u}{u^2+1}\,du}}_{\substack{w = u^2+1 \\ dw = 2u\,du}} \qquad \underbrace{}_{-7\,\text{TAN}^{-1}u}$

$\displaystyle\int \frac{1}{w}\,dw$

$= \ln|w|$

$= \ln(u^2+1)$

ANSWER:

$\ln\left((x+1)^2+1\right)$

$\quad - 7\,\text{TAN}^{-1}(x+1) + C$

4. (2 points) Evaluate the indefinite integral: $\displaystyle\int \frac{3}{2\sqrt{x}(1+x)}\,dx$

$u = \sqrt{x}$

$du = \frac{1}{2\sqrt{x}}\,dx$

$\displaystyle\int \frac{3}{1+u^2}\,du = 3\,\text{TAN}^{-1}u + C$

$= \boxed{3\,\text{TAN}^{-1}\sqrt{x} + C}$