Math 172 - Quiz 5

September 28, 2016

Name key

Score _____

Show all work to receive full credit. Supply explanations when necessary.

1. (1.5 points) Use the definitions of the hyperbolic functions to prove that

$$\sinh 2x + \cosh 2x = e^{2x}.$$

$\sinh 2x + \cosh 2x$

$= \frac{e^{2x} - e^{-2x}}{2} + \frac{e^{2x} + e^{-2x}}{2}$

$= \frac{e^{2x} + e^{2x}}{2} = e^{2x}$

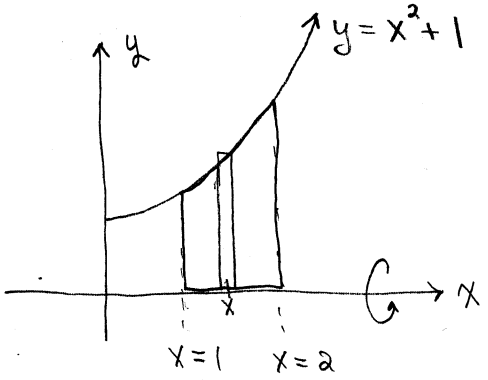
2. (1.5 points) Evaluate the integral by converting to exponentials: $\int \cosh 5x\, dx$

$$\int \frac{e^{5x} + e^{-5x}}{2}\, dx = \frac{1}{2}\int e^{5x} + e^{-5x}\, dx$$

$$= \frac{1}{2}\left[\frac{1}{5}e^{5x} - \frac{1}{5}e^{-5x}\right] + C$$

$$= \frac{1}{10}\left(e^{5x} - e^{-5x}\right) + C = \frac{1}{5}\sinh 5x + C$$

3. (3 points) The region bounded by the graphs of $y = x^2 + 1$, $x = 1$, $x = 2$, and $y = 0$ is rotated about the x-axis to form a solid. Find the volume of the solid.



$$\text{Volume} = \pi\int_1^2 (x^2+1)^2\, dx$$

$$= \pi\int_1^2 (x^4 + 2x^2 + 1)\, dx$$

$$= \pi\left(\frac{1}{5}x^5 + \frac{2}{3}x^3 + x\right)\Big|_1^2$$

$$= \pi\left(\frac{32}{5} + \frac{16}{3} + 2 - \frac{1}{5} - \frac{2}{3} - 1\right) = \frac{178\pi}{15} \approx 11.87\pi$$

4. (4 points) Find the total area of the bounded regions between the graphs of $y = x^4 - 9x^2$ and $y = x^3 - 9x$.

$x^4 - 9x^2 = x^2(x-3)(x+3)$

$x^3 - 9x = x(x-3)(x+3)$

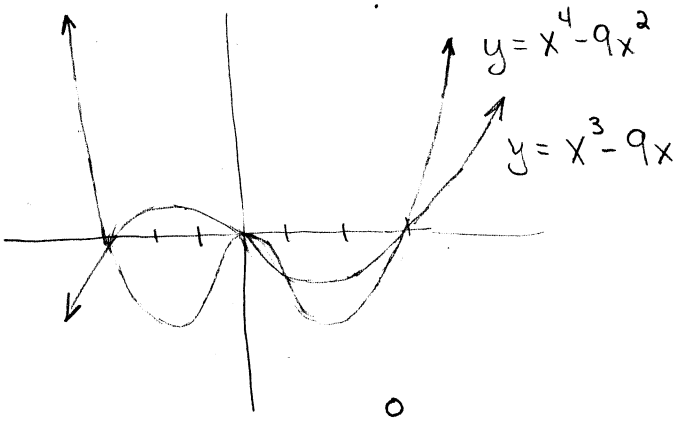
$x^4 - 9x^2 = x^3 - 9x$

$x^4 - x^3 - 9x^2 + 9x = 0$

$x(x^3 - x^2 - 9x + 9) = 0$

$x[x^2(x-1) - 9(x-1)] = 0$

$x(x-1)(x-3)(x+3) = 0$



$$\text{AREA} = \int_{-3}^{0} (x^3 - 9x - x^4 + 9x^2)\,dx - \int_{0}^{1} (x^3 - 9x - x^4 + 9x^2)\,dx + \int_{1}^{3} (x^3 - 9x - x^4 + 9x^2)\,dx$$

$$= \left(\frac{1}{4}x^4 - \frac{9}{2}x^2 - \frac{1}{5}x^5 + 3x^3\right)\Big|_{-3}^{0} - (\cdots)\Big|_{0}^{1} + (\cdots)\Big|_{1}^{3}$$

$$= 0 - \left(\frac{81}{4} - \frac{81}{2} + \frac{243}{5} - 81\right) - \left(\frac{1}{4} - \frac{9}{2} - \frac{1}{5} + 3\right) + 0 + \left(\frac{81}{4} - \frac{81}{2} - \frac{243}{5} + 81\right) - \left(\frac{1}{4} - \frac{9}{2} - \frac{1}{5} + 3\right)$$

$$= \boxed{\frac{677}{10}}$$