

Math 172 - Quiz 9

November 2, 2016

Name key

Score _____

Show all work to receive full credit. Supply explanations when necessary.

1. (3 points) Use integration by parts to evaluate the integral $\int \sin 3t \cos 7t dt$.

SIGNS	u & DERIVS	v' & ANTIS
+	$\sin 3t$	$\cos 7t$
-	$3 \cos 3t$	$\frac{1}{7} \sin 7t$
+	$-9 \sin 3t$	$-\frac{1}{49} \cos 7t$

$$\int \sin 3t \cos 7t dt = \frac{1}{7} \sin 3t \sin 7t + \frac{3}{49} \cos 3t \cos 7t + \frac{9}{49} \int \sin 3t \cos 7t dt$$

⇓

$$\int \sin 3t \cos 7t dt = \frac{49}{40} \left(\frac{1}{7} \sin 3t \sin 7t + \frac{3}{49} \cos 3t \cos 7t \right) + C$$

2. (3 points) Integrate $\int \frac{\cos^5 x}{\sqrt{\sin x}} dx$.

$$\int \frac{\cos^4 x \cos x}{\sqrt{\sin x}} dx = \int \frac{(1 - \sin^2 x)^2}{\sqrt{\sin x}} \cos x dx = \int \frac{(1 - u^2)^2}{\sqrt{u}} du$$

$$u = \sin x$$

$$du = \cos x dx$$

$$= \int (u^{-1/2} - 2u^{3/2} + u^{7/2}) du = 2u^{1/2} - \frac{4}{5} u^{5/2} + \frac{2}{9} u^{9/2} + C$$

$$= 2(\sin x)^{1/2} - \frac{4}{5} (\sin x)^{5/2} + \frac{2}{9} (\sin x)^{9/2} + C$$

3. (4 points) Evaluate $\int \frac{dx}{(x^2 - 1)^{3/2}}$ assuming $x < -1$.

$$x = \sec \theta, \quad \frac{\pi}{2} < \theta \leq \pi \quad (x < -1)$$

$$dx = \sec \theta \tan \theta d\theta$$

$$\int \frac{\sec \theta \tan \theta}{(\sec^2 \theta - 1)^{3/2}} d\theta = \int \frac{\sec \theta \tan \theta}{(\tan^2 \theta)^{3/2}} d\theta = \int \frac{\sec \theta \tan \theta}{|\tan \theta|^3} d\theta.$$

$$|\tan \theta| = -\tan \theta$$

$$\frac{\pi}{2} < \theta \leq \pi$$

$$= - \int \frac{\sec \theta}{\tan^2 \theta} d\theta$$

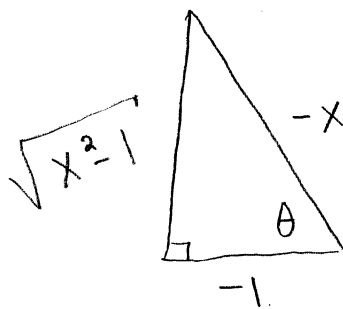
$$= - \int \frac{\cos \theta}{\sin^2 \theta} d\theta = - \int u^{-2} du$$

$$u = \sin \theta$$

$$du = \cos \theta d\theta$$

$$= u^{-1} + C$$

$$= \csc \theta + C$$



$$= \frac{-x}{\sqrt{x^2 - 1}} + C$$