Math 172 - Test 2

October 19, 2016

Name key

Score

Show all work to receive full credit. Supply explanations where necessary. Unless otherwise stated, you may use your calculator to evaluate any **definite** integrals.

1. (6 points) Use the definitions of the hyperbolic functions to evaluate all six functions at $x = 0$. Do not use your calculator.

$$\sinh 0 = \frac{e^0 - e^{-0}}{2} = \frac{1-1}{2} = 0 \qquad \operatorname{csch} 0 = \frac{1}{\sinh 0} = \text{DNE}$$

$$\cosh 0 = \frac{e^0 + e^{-0}}{2} = \frac{1+1}{2} = 1 \qquad \operatorname{sech} 0 = \frac{1}{\cosh 0} = 1$$

$$\tanh 0 = \frac{\sinh 0}{\cosh 0} = 0 \qquad \coth 0 = \frac{\cosh 0}{\sinh 0} = \text{DNE}$$

2. (8 points) Using the definitions of the hyperbolic functions (in terms of exponential functions), show that $\int \tanh x\, dx = \ln \cosh x + C$. (Hint: When integrating, you'll need to use a simple substitution.)

$$\int \tanh x \, dx = \int \frac{e^x - e^{-x}}{e^x + e^{-x}} \, dx$$

$$u = e^x + e^{-x}$$

$$du = (e^x - e^{-x})\, dx$$

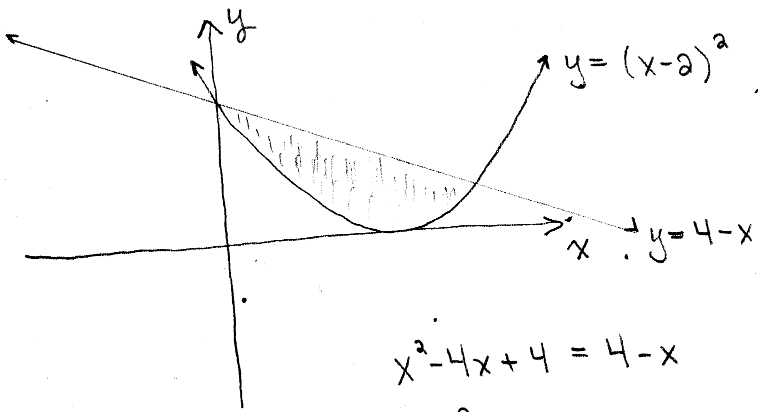
$$= \int \frac{1}{u}\, du = \ln|u| + C =$$

$$\ln(e^x + e^{-x}) + C$$

$2\cosh x = e^x + e^{-x}$

$$= \ln(\cosh x) + \underbrace{\ln 2 + C}_{\text{JUST A C}}$$

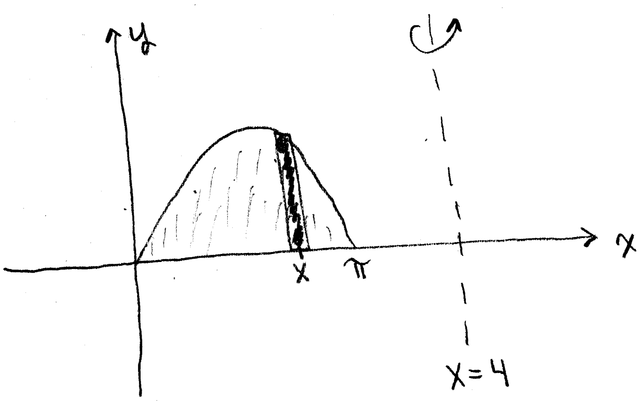
3. (12 points) Find the area of the region bounded by the graphs of $\overbrace{y = x^2 - 4x + 4}^{y=(x-2)^2}$ and $y = 4 - x$. Evaluate the definite integral(s) by hand.



$$x^2 - 4x + 4 = 4 - x$$
$$x^2 - 3x = 0$$
$$x = 0,\ x = 3$$

$$\text{AREA} = \int_0^3 (3x - x^2)\,dx$$
$$= \frac{3}{2}x^2 - \frac{1}{3}x^3 \Big|_0^3$$
$$= \frac{27}{2} - \frac{27}{3}$$
$$= \boxed{\frac{9}{2}}$$

4. (8 points) The region bounded by the graphs of $y = \sin x$, $y = 0$, $x = 0$, and $x = \pi$ is rotated about the line $x = 4$ to generate a solid of revolution. Find the volume of the solid. (Use your calculator to evaluate the required integral(s).)

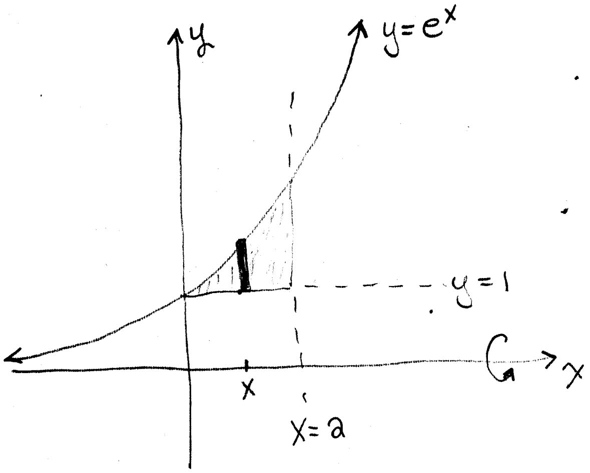


SHELLS...

$$2\pi \int_0^{\pi} (4-x)\sin x\,dx$$

$$\approx 30.526$$

5. (12 points) The 1st-quadrant region bounded by the graphs of $y = e^x$, $y = 1$, and $x = 2$ is rotated about the x-axis to generate a solid of revolution. Find the volume of the solid. Evaluate the definite integral(s) by hand.



WASHERS...

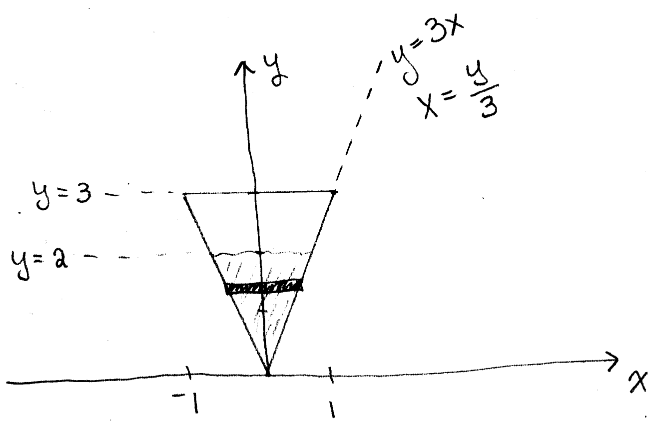
$$\pi \int_0^2 (e^x)^2 - (1)^2 \, dx$$

$$= \pi \int_0^2 (e^{2x} - 1) \, dx$$

$$= \pi \left(\frac{1}{2} e^{2x} - x \right)\Big|_0^2$$

$$= \boxed{\pi \left[\frac{1}{2} e^4 - 2 - \frac{1}{2} \right] \approx 77.91}$$

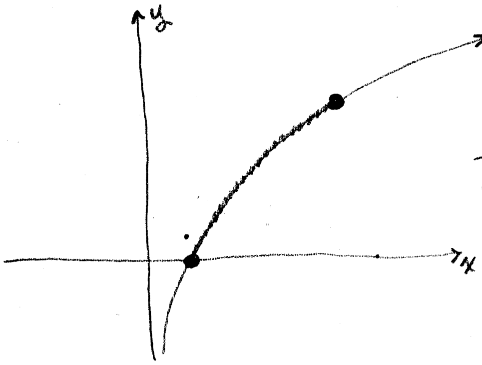
6. (10 points) A metal plate has the shape of an isosceles triangle with base length 2 feet and altitude 3 feet. The plate will be used as the end of a trough that will hold untreated sewage weighing 97 lbs/ft^3. Find the fluid force on the plate when the sewage in the trough is 2 feet deep. (Use your calculator to evaluate the required integral(s).)



$$97 \int_0^2 (2-y)(2)\left(\frac{y}{3}\right) dy$$

$$\approx \boxed{86.2 \text{ lbs}}$$

7. (6 points) A particle moves along the graph of $y = \ln x$ from the point $(1, 0)$ to the point $(e, 1)$. Find the distance traveled by the particle. (You may use your calculator to evaluate the required integral.)



$y = \ln x$

$\frac{dy}{dx} = \frac{1}{x}$.

Arc Length

$$= \int_1^e \sqrt{1 + \frac{1}{x^2}}\, dx$$

$$\approx 2.0035.$$

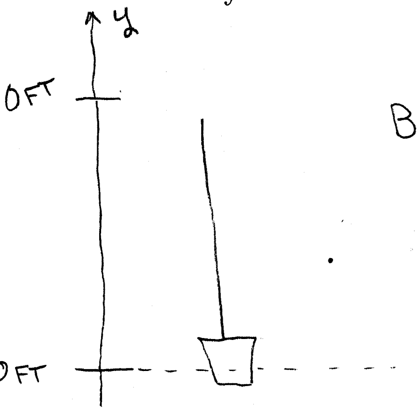
8. (12 points) A thin rod lies along the x-axis extending from $x = 3$ to $x = 8$. The density of the rod at x is given by $\rho(x) = 2x - 5$. Find the center of mass of the rod. (Use your calculator to evaluate the required integral(s).)

$$\text{Mass} = \int_3^8 (2x-5)\, dx = 30$$

$$M_0 = \int_3^8 x(2x-5)\, dx = \frac{1115}{6}$$

$$\text{C.M.} = \frac{1115/6}{30} = \boxed{\frac{223}{36} \approx 6.19}$$

9. (10 points) A weightless rope is used to lift a leaky bucket of water 20 ft up to the top of a roof. The bucket initially weighs 38 lbs, and by the time it reaches the roof, it has lost 28 lbs. Find the work done in lifting the bucket. Evaluate the definite integral(s) by hand.



BUCKET STARTS AT 38 lbs

AND LOSES $\frac{28}{20} = 1.4$ lb PER FT.

$F(y)$ = WEIGHT OF BUCKET AT y

$= 38 - 1.4y$

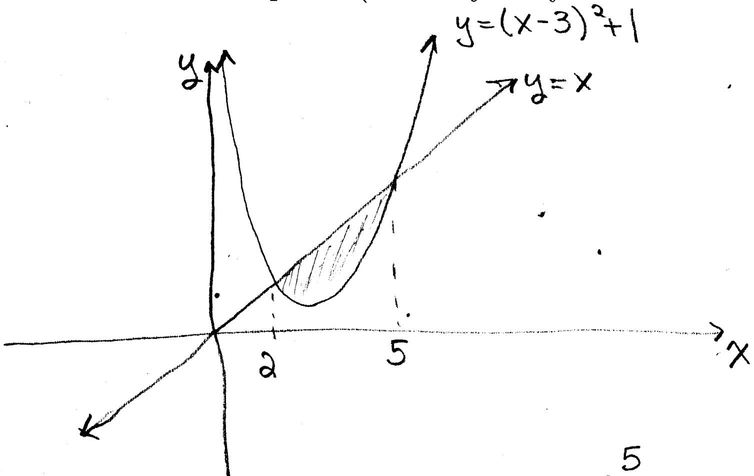
$$\text{TOTAL WORK} = \int_0^{20} (38 - 1.4y)\,dy$$

$$= 38y - 0.7y^2 \Big|_0^{20}$$

$$= 38(20) - 0.7(400)$$

$$= \text{480 FT-LBS}$$

10. (16 points) A thin plate is bounded by the graphs of $y = x$ and $y = (x-3)^2 + 1$. The density of the plate at the point (x, y) is given by $\rho(x) = \sqrt{x} - 1$. Find the center of mass of the plate. (You may use your calculator to evaluate the required integrals.)



$$(x-3)^2 + 1 = x$$
$$x^2 - 7x + 10 = 0$$
$$(x-2)(x-5) = 0$$
$$x = 2,\ x = 5$$

$$\text{MASS} = \int_2^5 (\sqrt{x}-1)\left[x - (x-3)^2 - 1\right] dx$$

$$\approx 3.879$$

$$M_y = \int_2^5 x(\sqrt{x}-1)\left[x - (x-3)^2 - 1\right] dx$$

$$\approx 14.124$$

$$M_x = \int_2^5 \left(\frac{x + (x-3)^2 + 1}{2}\right)(\sqrt{x}-1)\left[x - (x-3)^2 - 1\right] dx$$

$$\approx 10.622$$

$$CM = \left(\frac{M_y}{M}, \frac{M_x}{M}\right) \approx \boxed{(3.641,\ 2.738)}$$