Math 172 - Quiz 12

November 29, 2017

Name Key

Score _____

Show all work to receive full credit. Supply explanations when necessary.

1. (4 points) Write the first five terms of the sequence whose nth term is given.

   (a) $a_n = \dfrac{3n}{n+4}$ $\quad \left\{ \frac{3}{5}, \frac{6}{6}, \frac{9}{7}, \frac{12}{8}, \frac{15}{9}, \ldots \right\}$

   (b) $a_n = 4 + \cos \pi n$ $\quad \{ 3, 5, 3, 5, 3, \ldots \}$

   (c) $a_n = n \sin \dfrac{1}{n}$ $\quad \left\{ \sin 1, 2 \sin \frac{1}{2}, 3 \sin \frac{1}{3}, 4 \sin \frac{1}{4}, 5 \sin \frac{1}{5}, \ldots \right\}$

2. (4 points) Determine whether each sequence above converges or diverges. If it converges, find the limit.

a) $\displaystyle\lim_{x\to\infty} \frac{3x}{x+4} = \boxed{3}$

b) $\displaystyle\lim_{x\to\infty} (4 + \cos \pi x)$

$\displaystyle = 4 + \lim_{x\to\infty} \cos \pi x$ $\quad \boxed{\text{DNE}}$

c) $\displaystyle\lim_{x\to\infty} x \sin \frac{1}{x}$ $\quad \infty \cdot 0$

$$= \lim_{x\to\infty} \frac{\sin \frac{1}{x}}{\frac{1}{x}} = \lim_{x\to\infty} \frac{-\cancel{\frac{1}{x^2}} \cos \frac{1}{x}}{-\cancel{\frac{1}{x^2}}} = \cos 0 = \boxed{1}$$

$0/0$

L'HÔPITAL'S RULE

3. (2 points) Write the first five terms of the recursively defined sequence.

$$a_0 = 1; \quad a_1 = 1; \quad a_{n+1} = a_n + a_{n-1},\ n = 1, 2, 3, \ldots$$

$1, 1, 2, 3, 5, 8, 13, \ldots$