Math 172 - Quiz 4

September 13, 2017

Name key

Score _____

Show all work to receive full credit. Supply explanations when necessary.

1. (3 points) Find the slope of the line tangent to the graph of $y = x\,7^{x^2}$ at the point where $x = 1$.

$$\frac{dy}{dx} = 7^{x^2} + x\underbrace{\frac{d}{dx}7^{x^2}} = 7^{x^2} + 2x^2 \ln 7\, e^{x^2 \ln 7}$$
$$= 7^{x^2}\left[1 + 2x^2 \ln 7\right]$$

$$\frac{d}{dx} e^{x^2 \ln 7} = e^{x^2 \ln 7}(2x \ln 7)$$

WHEN $x = 1$,

$$\frac{dy}{dx} = m = 7(1 + 2\ln 7) \approx 34.24$$

2. (3 points) Integrate: $\displaystyle\int \frac{\log_3 x^2}{5x}\,dx$

$$\frac{2}{5\ln 3}\int \frac{\ln x}{x}\,dx = \frac{2}{5\ln 3}\int u\,du$$

$u = \ln x$

$du = \frac{1}{x}dx$

$$= \frac{1}{5\ln 3}u^2 + C = \frac{1}{5\ln 3}(\ln x)^2 + C$$

3. (3 points) Determine the derivative of $y = x^x$.

$$\ln y = x \ln x$$
$$\frac{1}{y}\frac{dy}{dx} = \ln x + x\left(\frac{1}{x}\right)$$
$$\frac{dy}{dx} = y(\ln x + 1)$$

$$\frac{dy}{dx} = x^x(1 + \ln x)$$

4. (1 point) What is the exact value of $\underbrace{\sin^{-1}(\sin(13\pi/6))}$?

$$\sin^{-1}\left(\sin\left(\frac{\pi}{6}\right)\right) = \frac{\pi}{6}$$