

Math 173 - Homework

April 8, 2010

Name key

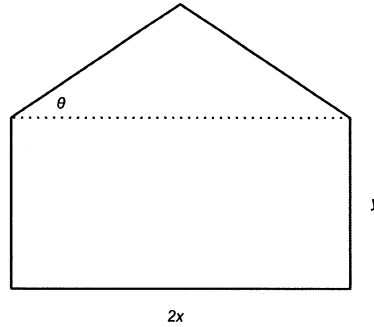
Score _____

Show all work to receive full credit. Supply explanations when necessary. This problem is worth 5 points and is due on Wednesday, April 14. You may use a computer algebra system for any part of the problem.

The pentagon shown below has a fixed perimeter of 100 cm. Find x , y , and θ so that the area is a maximum.

$$\text{MAXIMIZE } 2xy + x^2 \tan \theta$$

$$\text{s.t. } \underbrace{2x + 2y + 2x \sec \theta}_{x + y + x \sec \theta} = 100 = 50$$



LAGRANGE MULTIPLIERS:

$$2y + 2x \tan \theta = \lambda (1 + \sec \theta)$$

$$2x = \lambda$$

$$x^2 \sec^2 \theta = \lambda x \sec \theta \tan \theta$$

$$x + y + x \sec \theta = 50$$

$$x^2 \sec^2 \theta = 2x^2 \sec \theta \tan \theta$$

$\sec \theta \neq 0$ AND $x = 0$ IS NO GOOD,

SO WE MUST HAVE

$$\sec \theta = 2 \tan \theta$$

$$\text{OR } \sin \theta = \frac{1}{2} \Rightarrow \theta = \frac{\pi}{6}$$

Top & Bottom

EQUATIONS NOW GIVE:

$$2y + \frac{2}{\sqrt{3}} x = 2x \left(1 + \frac{2}{\sqrt{3}}\right)$$

$$x + y + \frac{2}{\sqrt{3}} x = 50$$

LINEAR SYSTEM IN x & y .

SOLUTION IS

$$x = 100 - 50\sqrt{3}$$

$$y = 50 - \frac{50}{\sqrt{3}}$$

$$\theta = \frac{\pi}{6}, \quad x = 100 - 50\sqrt{3}, \quad y = 50 - \frac{50}{\sqrt{3}}$$

THESE GIVE AN AREA OF

$$5000 - 2500\sqrt{3}$$

$$\approx 669.87$$

THIS MUST BE A MAX SINCE

$x = 10, y = 30, \theta = 0$ SATISFY THE CONSTRAINT AND GIVE A SMALLER AREA (I.E. 600).