Math 173 - Quiz 1

January 28, 2010

Name key

Score _____

Show all work to receive full credit. Supply explanations when necessary.

1. (4 points) Find the 4th Taylor polynomial for $f(x) = \frac{1}{x}$ centered at $x = 1$.

$f(x) = \frac{1}{x}$, $f(1) = 1$

$f'(x) = -\frac{1}{x^2}$, $f'(1) = -1$

$f''(x) = \frac{2}{x^3}$, $f''(1) = 2$

$f'''(x) = -\frac{6}{x^4}$, $f'''(1) = -6$

$f^{(4)}(x) = \frac{24}{x^5}$, $f^{(4)}(1) = 24$

$P_4(x) = 1 - 1(x-1) + \frac{2}{2!}(x-1)^2 - \frac{6}{3!}(x-1)^3 + \frac{24}{4!}(x-1)^4$

$P_4(x) = 1 - (x-1) + (x-1)^2 - (x-1)^3 + (x-1)^4$

2. (2 points) Use your calculator to find the 3rd Maclaurin polynomial for $f(x) = \dfrac{\sin x}{x+3}$. Write down your result and use it to approximate $f(0.125)$. Round your answer to the 7th decimal place.

$f(x) = \frac{\sin x}{x+3}$, $P_3(x) = \frac{x}{3} - \frac{x^2}{9} - \frac{x^3}{54}$

$P_3(0.125) = 0.0398944$

3. (4 points) Determine the interval of convergence of the power series $\sum_{n=1}^{\infty} \dfrac{(x-2)^n}{n^2 5^n}$.

RATIO TEST:

$$\lim_{N\to\infty} \frac{|x-2|^{N+1}}{(N+1)^2 5^{N+1}} \cdot \frac{N^2 5^N}{|x-2|^N}$$

$$= \lim_{N\to\infty} \frac{N^2}{(N+1)^2} \frac{|x-2|}{5}$$

$$= \frac{|x-2|}{5} \lim_{N\to\infty} \frac{N^2}{(N+1)^2} = \frac{|x-2|}{5}(1)$$

$\frac{|x-2|}{5} < 1 \Rightarrow |x-2| < 5$

$\Rightarrow -5 < x-2 < 5$

$-3 < x < 7$

ENDPOINTS:

$x = -3 \ldots \sum \frac{(-5)^N}{N^2 5^N} = \sum \frac{(-1)^N}{N^2}$

CONVERGES ABSOLUTELY, $p = 2$ SERIES

$x = 7 \ldots \sum \frac{5^N}{N^2 5^N} = \sum \frac{1}{N^2}$

CONVERGES, $p = 2$ SERIES

INTERVAL OF CONVERGENCE IS $[-3, 7]$