Math 173 - Quiz 3

February 11, 2010

Name key

Score _____

Show each step to receive full credit. Supply explanations when necessary.

1. (3 points) Let $\vec{u} = 3\hat{\imath} - 4\hat{\jmath} + 7\hat{k}$ and let $\vec{v} = -5\hat{\imath} - \hat{\jmath} + 3\hat{k}$. Find $\vec{v} \times \vec{u}$.

$$\vec{v} \times \vec{u} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ -5 & -1 & 3 \\ 3 & -4 & 7 \end{vmatrix} = \hat{\imath}(-7+12) - \hat{\jmath}(-35-9) + \hat{k}(20+3)$$

$$= \boxed{5\hat{\imath} + 44\hat{\jmath} + 23\hat{k}}$$

2. (3 points) The points P, Q, and R are given below. Find the angle between the line segments $\overline{PQ}$ and $\overline{PR}$.

$P(2,2,2)$ $\quad Q(5,3,9)$ $\quad R(-4,6,4)$

$$\vec{PQ} = 3\hat{\imath} + \hat{\jmath} + 7\hat{k}$$

$$\vec{PR} = -6\hat{\imath} + 4\hat{\jmath} + 2\hat{k}$$

$$\cos\theta = \frac{\vec{PQ} \cdot \vec{PR}}{|\vec{PQ}|\,|\vec{PR}|} = \frac{-18 + 4 + 14}{|\vec{PQ}|\,|\vec{PR}|} = 0$$

$$\Rightarrow \boxed{\theta = 90^\circ}$$

3. (3 points) Find the projection of $\vec{u} = 2\hat{\imath} + \hat{\jmath} + \hat{k}$ onto $\vec{v} = -3\hat{\imath} + 2\hat{\jmath} - 2\hat{k}$.

$$\text{proj}_{\vec{v}}\,\vec{u} = \frac{\vec{u} \cdot \vec{v}}{\vec{v} \cdot \vec{v}}\,\vec{v} \; ; \qquad \vec{u} \cdot \vec{v} = -6 + 2 - 2 = -6$$

$$\vec{v} \cdot \vec{v} = 9 + 4 + 4 = 17$$

$$\text{proj}_{\vec{v}}\,\vec{u} = -\frac{6}{17}(-3\hat{\imath} + 2\hat{\jmath} - 2\hat{k}) = \boxed{\frac{18}{17}\hat{\imath} - \frac{12}{17}\hat{\jmath} + \frac{12}{17}\hat{k}}$$

4. (1 point) If $\vec{u}$ and $\vec{v}$ are orthogonal to $\vec{w}$, is $\vec{u} + \vec{v}$ orthogonal to $\vec{w}$? Explain your reasoning.

YES, IF $\vec{u} \cdot \vec{w} = 0$ AND $\vec{v} \cdot \vec{w} = 0$, THEN

$$(\vec{u} + \vec{v}) \cdot \vec{w} = \vec{u} \cdot \vec{w} + \vec{v} \cdot \vec{w} = 0 + 0 = 0.$$