Math 173 - Quiz 4

February 25, 2010

Name key

Score ______

Show each step to receive full credit. Supply explanations when necessary.

1. (1 point) Without actually carrying out any computations, tell whether the point $(\rho, \theta, \phi) = (3, \pi/6, 2\pi/3)$ lies above or below the xy-plane. Explain.

   SINCE $\varphi = \frac{2\pi}{3} > \frac{\pi}{2}$, THIS POINT LIES BELOW THE XY-PLANE.

2. (2 points) Convert the point $(x, y, z) = (-3, 4, -5)$ to spherical coordinates.

   $\rho^2 = (-3)^2 + (4)^2 + (-5)^2$

   $\rho^2 = 9 + 16 + 25$

   $\rho^2 = 50, \quad \rho = 5\sqrt{2}$

   $\tan\theta = -\frac{4}{3}$ AND θ IS IN QUAD 2.

   $\theta = 180° + \tan^{-1}\left(-\frac{4}{3}\right)$

   $\theta \approx 126.87°$

   $\varphi = \cos^{-1}\left(\frac{-5}{5\sqrt{2}}\right) = 135°$

   $(5\sqrt{2},\ 126.87°,\ 135°)$

3. (2 points) What is the domain of the following vector-valued function?

   $$\vec{r}(t) = \underbrace{\frac{\ln t}{t^2+1}}_{t>0}\hat{\imath} + \underbrace{\frac{e^t}{t}}_{t\neq 0}\hat{\jmath} + \underbrace{\sqrt{t+1}}_{t\geq -1}\hat{k}$$

   DOMAIN $= \{t : t > 0\}$

4. (2 points) Find a unit vector in the direction of $\vec{r}(t) = (\cos t)\hat{\imath} - (\sin t)\hat{\jmath} + t\hat{k}$.

   $|\vec{r}(t)| = \sqrt{\cos^2 t + \sin^2 t + t^2}$

   $= \sqrt{1 + t^2}$

   $\frac{\vec{r}(t)}{|\vec{r}(t)|} = \frac{1}{\sqrt{1+t^2}}\left(\cos t\,\hat{\imath} - \sin t\,\hat{\jmath} + t\,\hat{k}\right)$

5. (3 points) Suppose $\vec{r}'(t) = (\sin 2t)\hat{\imath} + t\hat{\jmath} + e^{2t}\hat{k}$. Find $\vec{r}(t)$ if $\vec{r}(0) = \hat{\imath} + \hat{\jmath} + \hat{k}$.

   $\vec{r}(t) = \left(-\frac{1}{2}\cos 2t + c_1\right)\hat{\imath} + \left(\frac{t^2}{2} + c_2\right)\hat{\jmath} + \left(\frac{1}{2}e^{2t} + c_3\right)\hat{k}$

   $\vec{r}(0) = \left(-\frac{1}{2} + c_1\right)\hat{\imath} + c_2\hat{\jmath} + \left(\frac{1}{2} + c_3\right)\hat{k} = \hat{\imath} + \hat{\jmath} + \hat{k} \Rightarrow c_1 = \frac{3}{2},\ c_2 = 1,\ c_3 = \frac{1}{2}$

   $\vec{r}(t) = \left(-\frac{1}{2}\cos 2t + \frac{3}{2}\right)\hat{\imath} + \left(\frac{t^2}{2} + 1\right)\hat{\jmath} + \left(\frac{1}{2}e^{2t} + \frac{1}{2}\right)\hat{k}$