Math 173 - Quiz 5

March 4, 2010

Name key

Score

Show all work to receive full credit. Supply explanations when necessary.

1. (2 points) Find a set of parametric equations for the line tangent to the graph of $\vec{r}(t)$ at the given point.

$$\vec{r}(t) = t\hat{\imath} + t^2\hat{\jmath} - 3t\hat{k}, \quad P(0,0,0)$$

$\vec{r}\,'(t) = \hat{\imath} + 2t\hat{\jmath} - 3\hat{k}$

$(0,0,0)$ CORRESPONDS TO $t = 0$

$\vec{r}\,'(0) = \hat{\imath} - 3\hat{k}$ = DIRECTION VECTOR

$(0,0,0)$ = POINT

$$\boxed{\begin{aligned} x &= t \\ y &= 0 \\ z &= -3t \end{aligned}}$$

2. (5 points) To open the 1992 Summer Olympics in Barcelona, bronze medalist archer Antonio Rebollo lit the Olympic torch with a flaming arrow. Suppose that Rebollo shot the arrow at a height of 6 ft above the ground 90 ft from a 70-ft-high cauldron, and he wanted the arrow to reach its maximum height exactly 4 ft above the center of the cauldron. Find the initial speed and firing angle of the arrow.

$\vec{r}(t) = V_0 \cos\theta\, t\, \hat{\imath} + (-16t^2 + V_0 \sin\theta\, t + 6)\hat{\jmath}$

SIMULTANEOUSLY, WE MUST HAVE

$V_0 \cos\theta\, t = 90$

$-16t^2 + V_0 \sin\theta\, t + 6 = 74$

$-32t + V_0 \sin\theta = 0$

$-16t^2 + 32t^2 = 68$

$t = 2.06155$

$$\frac{V_0 \sin\theta\, t = 68 + 16t^2}{V_0 \cos\theta\, t = 90} \Rightarrow \tan\theta = \frac{136}{90} \Rightarrow \boxed{\theta = 56.5^\circ}$$

$V_0 = \dfrac{90}{\cos\theta\, t} = \boxed{79.1 \text{ FT/SEC}}$

3. (3 points) Find the unit tangent vector for following vector-valued function.

$$\vec{r}(t) = (e^t \cos t)\hat{\imath} + (e^t \sin t)\hat{\jmath} + 2\hat{k}$$

$\vec{r}\,'(t) = (e^t \cos t - e^t \sin t)\hat{\imath} + (e^t \sin t + e^t \cos t)\hat{\jmath}$

$|\vec{r}\,'(t)| = \left[e^{2t}\cos^2 t - 2e^{2t}\sin t \cos t + e^{2t}\cos^2 t + e^{2t}\sin^2 t + 2e^{2t}\sin t\cos t + e^{2t}\cos^2 t\right]^{1/2}$

$= \sqrt{2e^{2t}} = \sqrt{2}\,e^t$

$$\boxed{\hat{T}(t) = \frac{1}{\sqrt{2}}\left[(\cos t - \sin t)\hat{\imath} + (\sin t + \cos t)\hat{\jmath}\right]}$$