Math 173 - Quiz 6

April 1, 2010

Name key

Score _____

Show all work to receive full credit. Supply explanations when necessary.

1. (2 points) Use the chain rule to find a formula for $\partial w/\partial t$.

$$w = x^2 e^{xy} \qquad x = s\cos t \qquad y = s^2 + t^2$$

$$\frac{\partial w}{\partial t} = \frac{\partial w}{\partial x}\frac{\partial x}{\partial t} + \frac{\partial w}{\partial y}\frac{\partial y}{\partial t}$$

$$= \boxed{\left(2x e^{xy} + x^2 y e^{xy}\right)\left(-s\sin t\right) + \left(x^3 e^{xy}\right)\left(2t\right)}$$

2. (3 points) Suppose that z is implicitly defined as a function of x and y by the following equation. Find $\partial z/\partial y$ and $\partial z/\partial x$.

$$\underbrace{x^3 z + y^3 z^3 - 9xyz + 4 = 0}_{F(x,y,z)}$$

$$\frac{\partial z}{\partial y} = -\frac{F_y}{F_z} = \frac{-\left(3y^2 z^3 - 9xz\right)}{x^3 + 3y^3 z^2 - 9xy}$$

$$\frac{\partial z}{\partial x} = -\frac{F_x}{F_z} = \frac{-\left(3x^2 z - 9yz\right)}{x^3 + 3y^3 z^2 - 9xy}$$

3. (2 points) In 1897 Thomson conducted a landmark experiment in modern physics when he measured the charge-to-mass ratio of an electron. The currently accepted value of the electron's charge is $1.602176487 \times 10^{-19}$ C with an error of $\pm 0.000000040 \times 10^{-19}$ C. The electron's mass is $9.10938215 \times 10^{-31}$ kg, with an error of $\pm 0.00000045 \times 10^{-31}$ kg. Compute the electron's charge-to-mass ratio and use differentials to approximate the error.

$$z = \frac{q}{m}, \quad dz = \frac{1}{m}\,dq - \frac{q}{m^2}\,dm$$

$$\Delta z \approx \frac{1}{m}\,\Delta q - \frac{q}{m^2}\,\Delta m$$

Using numbers above, we get $\Delta z \approx -4297$

$$\frac{q}{m} = 1.7588820149 \times 10^{11} \pm \underbrace{4297}$$

or

$0.00000004297 \times 10^{11}$

4. (3 points) Use the definition of differentiability to show that $f(x, y) = 5x^2 - 10y + y^3$ is differentiable on $\mathbb{R}^2$.

$f_x(x,y) = 10x$, $f_y(x,y) = -10 + 3y^2$

$$\Delta z = f(x+\Delta x, y+\Delta y) - f(x,y) = 5(x+\Delta x)^2 - 10(y+\Delta y) + \underbrace{(y+\Delta y)^3}_{y^3 + 3y^2\Delta y + 3y\Delta y^2 + \Delta y^3} - (5x^2 - 10y + y^3)$$

$$= 10x\Delta x + 5\Delta x^2 - 10\Delta y + 3y^2\Delta y + 3y\Delta y^2 + \Delta y^3$$

$$= \underbrace{(10x)}_{f_x}\Delta x + \underbrace{(-10+3y^2)}_{f_y}\Delta y + \underbrace{(5\Delta x)}_{\epsilon_1}\Delta x + \underbrace{(3y\Delta y + \Delta y^2)}_{\epsilon_2}\Delta y$$

Since $(\epsilon_1, \epsilon_2) \to (0,0)$ as $(\Delta x, \Delta y) \to (0,0)$, f is differentiable.