Math 173 - Quiz 7

April 8, 2010

Name key

Score

Show all work to receive full credit. Supply explanations when necessary.

1. (4 points) Find equations of the tangent plane and the normal line (parametric and symmetric) at the point $(1, 0, 0)$.

$$z + 1 = xe^y \cos z$$

$$F(x,y,z) = xe^y \cos z - z = 1$$

$$\vec{\nabla} F(x,y,z) = e^y \cos z \,\hat{\imath} + xe^y \cos z \,\hat{\jmath} - (xe^y \sin z + 1)\hat{k}$$

$$\vec{\nabla} F(1,0,0) = \hat{\imath} + \hat{\jmath} - \hat{k}$$

Tangent Plane

$$1(x-1) + 1(y-0) - 1(z-0) = 0$$

$$\boxed{x + y - z = 1}$$

Normal Line

$$x = 1 + t$$
$$y = t$$
$$z = -t$$

$$x - 1 = y = -z$$

2. (2 points) Over a certain region in space the electrical potential V is given by

$$V(x, y, z) = 5x^2 - 3xy + xyz.$$

At the point $(3, 4, 5)$, in which direction does the potential increase most rapidly?

$$\vec{\nabla} V(x,y,z) = (10x - 3y + yz)\hat{\imath} + (-3x + xz)\hat{\jmath} + (xy)\hat{k}$$

$$\vec{\nabla} V(3,4,5) = (30 - 12 + 20)\hat{\imath} + (-9 + 15)\hat{\jmath} + 12\hat{k}$$

$$= \boxed{38\hat{\imath} + 6\hat{\jmath} + 12\hat{k}}$$

3. (4 points) Find and classify the critical points of $f(x,y) = x^3 - 2xy + xy^2 - 7$.

$f_x(x,y) = 3x^2 - 2y + y^2 = 0$

$f_y(x,y) = -2x + 2xy = 0 \Rightarrow 2x(-1+y) = 0$

$x = 0$ OR $y = 1$

$-2y + y^2 = 0$

$y(-2+y) = 0$

$y = 0$ OR $y = 2$

$3x^2 - 2 + 1 = 0$

$3x^2 = 1$

$x = \pm \frac{1}{\sqrt{3}}$

FOUR CRIT POINTS:

$(0,0),\ (0,2),\ \left(\frac{1}{\sqrt{3}}, 1\right),\ \left(-\frac{1}{\sqrt{3}}, 1\right)$

$$D(x,y) = \begin{vmatrix} 6x & 2y-2 \\ 2y-2 & 2x \end{vmatrix} = 12x^2 - (2y-2)^2$$

$D(0,0) = -4 < 0 \Rightarrow$ $(0,0,-7)$ IS A SADDLE PT.

$D(0,2) = -4 < 0 \Rightarrow$ $(0,2,-7)$ IS A SADDLE PT.

$D\left(\frac{1}{\sqrt{3}}, 1\right) = 4 > 0$ & $f_{xx}\left(\frac{1}{\sqrt{3}}, 1\right) > 0 \Rightarrow$ $f\left(\frac{1}{\sqrt{3}}, 1\right) \approx -7.385$ IS A REL MIN

$D\left(-\frac{1}{\sqrt{3}}, 1\right) = 4 > 0$ & $f_{xx}\left(-\frac{1}{\sqrt{3}}, 1\right) < 0 \Rightarrow$ $f\left(-\frac{1}{\sqrt{3}}, 1\right) \approx -6.615$ IS A REL MAX.