Math 173 - Quiz 8

April 15, 2010

Name key

Score _____

Show each step to receive full credit. Supply explanations when necessary.

1. (4 points) Use Lagrange multipliers to find the extreme values of $f(x,y) = y^2 - 4x$ subject to $x^2 + y^2 = 9$.

$\vec{\nabla} f(x,y) = -4\hat{i} + 2y\hat{j}$

$\vec{\nabla} g(x,y) = 2x\hat{i} + 2y\hat{j}$

$-4 = 2\lambda x$

$2y = 2\lambda y \longrightarrow 2y - 2\lambda y = 0$

$\Rightarrow 2y(1-\lambda) = 0$

$y = 0$ or $\lambda = 1$

$x^2 + y^2 = 9$

$y = 0$: $x^2 = 9$, $x = \pm 3$

$\lambda = 1$: $x = -2$, $y^2 = 5$, $y = \pm\sqrt{5}$

FOUR CRITICAL POINTS:

$(3,0)$, $(-3,0)$, $(-2,\sqrt{5})$, $(-2,-\sqrt{5})$

$f(3,0) = -12$ } MIN VALUE IS -12 AT $(3,0)$

$f(-3,0) = 12$

$f(-2,\sqrt{5}) = 13$ }

$f(-2,-\sqrt{5}) = 13$ } MAX VALUE IS 13 AT $(-2,\sqrt{5})$ AND $(-2,-\sqrt{5})$

2. (6 points) Consider the iterated integral $\int_0^4 \int_y^{2y} (8x + e^y)\, dx\, dy$.

(a) Evaluate the iterated integral. You'll need integration by parts. Show your work!

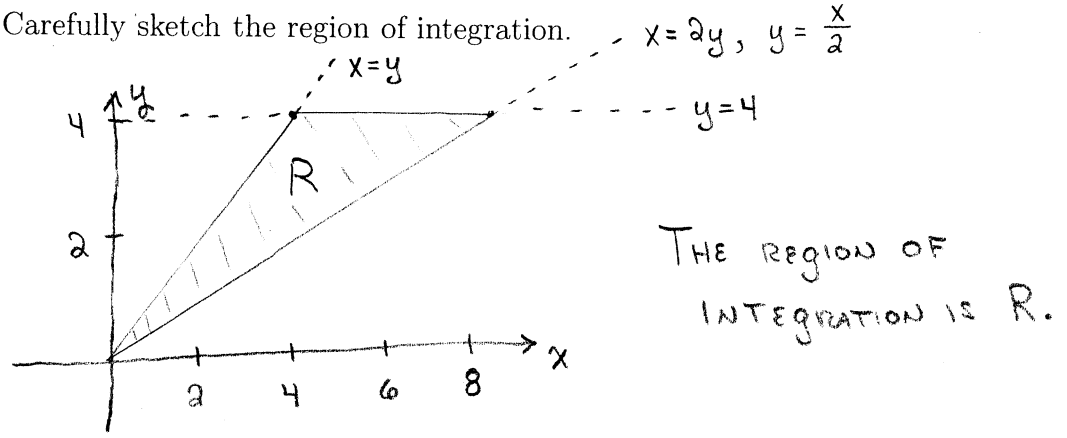
$$\int_0^4 \left(4x^2 + xe^y\right)\Big|_y^{2y} dy = \int_0^4 \left(12y^2 + ye^y\right) dy = 4y^3\Big|_0^4 + ye^y\Big|_0^4 - \int_0^4 e^y dy$$

$u = y \quad du = dy$

$dv = e^y dy \quad v = e^y$

$$= 4y^3\Big|_0^4 + ye^y\Big|_0^4 - e^y\Big|_0^4 = 256 + 3e^4 + 1 = \boxed{257 + 3e^4}$$

(b) Carefully sketch the region of integration.



THE REGION OF INTEGRATION IS R.

(c) Set up the iterated integrals with the reversed order of integration. You'll need two iterated integrals.

$$\int_{x=0}^{x=4} \int_{y=x/2}^{y=x} (8x + e^y)\, dy\, dx + \int_{x=4}^{x=8} \int_{y=x/2}^{y=4} (8x + e^y)\, dy\, dx$$

$$e^4 - 2e^2 + \frac{259}{3} + 2e^4 + 2e^2 + \frac{512}{3}$$

$$= 3e^4 + 257$$