Math 173 - Quiz 5

March 10, 2011

Name key

Score

Show all work to receive full credit. Supply explanations when necessary.

1. (2 points) Show that the following limit does not exist.

$$\lim_{(x,y)\to(0,0)} \frac{xy^2}{x^2+y^4}$$

Along $x = 0$: $\lim_{y\to 0} \frac{0}{y^4} = \lim_{y \to 0} 0 = 0$

Along $x = y^2$: $\lim_{y\to 0} \frac{y^4}{2y^4} = \lim_{y\to 0} \frac{1}{2} = \frac{1}{2}$

Two different limits along two paths.

Limit DNE.

2. (3 points) Show that $f(x, y) = x^2 + 3xy + y^2$ is differentiable everywhere on $\mathbb{R}^2$.

$$\Delta z = f(x+\Delta x, y+\Delta y) - f(x,y) = (x+\Delta x)^2 + 3(x+\Delta x)(y+\Delta y) + (y+\Delta y)^2 - x^2 - 3xy - y^2$$

$$= x^2 + \underline{2x\Delta x} + \underline{\underline{\Delta x^2}} + 3xy + \underline{3x\Delta y} + \underline{3y\Delta x} + \underline{\underline{3\Delta x\Delta y}} + y^2 + \underline{2y\Delta y} + \underline{\underline{\Delta y^2}} - x^2 - 3xy - y^2$$

$$= \underbrace{(2x+3y)}_{f_x(x,y)}\Delta x + \underbrace{(3x+2y)}_{f_y(x,y)}\Delta y + \underbrace{(\Delta x)}_{\epsilon_1}\Delta x + \underbrace{(3\Delta x + \Delta y)}_{\epsilon_2}\Delta y$$

Since Δz has the correct form and

ϵ_1 & $\epsilon_2 \to 0$ as $(\Delta x, \Delta y) \to (0,0)$

and this is true on all of $\mathbb{R}^2$,

f is differentiable everywhere.

3. (2 points) In 1897 Thomson conducted a landmark experiment in modern physics when he measured the charge-to-mass ratio of an electron. The currently accepted value of the electron's charge is $1.602176487 \times 10^{-19}$ C with an error of $\pm 0.000000040 \times 10^{-19}$ C. The electron's mass is $9.10938215 \times 10^{-31}$ kg, with an error of $\pm 0.00000045 \times 10^{-31}$ kg. Compute the electron's charge-to-mass ratio and use differentials to approximate the error.

$$z = \frac{q}{m}, \quad dz = \frac{1}{m}\,dq - \frac{q}{m^2}\,dm \Rightarrow \Delta z = \frac{1}{m}\Delta q - \frac{q}{m^2}\Delta m$$

USING THE NUMBERS ABOVE, $z = 1.75882015 \times 10^{11}$

AND $\Delta z = -4297.4272 \approx -0.000000043 \times 10^{11}$

So

$$\frac{q}{m} = \left(1.75882015 \times 10^{11}\right) \pm \left(0.000000043 \times 10^{11}\right)$$

4. (3 points) Suppose that z is implicitly defined as a function of x and y by the following equation. Find $\partial z/\partial y$ and $\partial z/\partial x$.

$$x^3 z + y^3 z^3 - 9xyz + 4 = 0$$

$$\frac{\partial z}{\partial x} = \frac{-f_x}{f_z}, \quad \frac{\partial z}{\partial y} = \frac{-f_y}{f_z}$$

$$\frac{\partial z}{\partial x} = \frac{-(3x^2 z - 9yz)}{x^3 + 3y^3 z^2 - 9xy}$$

$$\frac{\partial z}{\partial y} = \frac{-(3y^2 z^3 - 9xz)}{x^3 + 3y^3 z^2 - 9xy}$$