Math 173 - Quiz 6

March 31, 2011

Name key

Score

Show all work to receive full credit. Supply explanations when necessary.

1. (4.5 points) Find all relative extreme points and saddle points.

$$f(x, y) = -x^3 + 4xy - 2y^2 + 1$$

$f_x(x,y) = -3x^2 + 4y = 0$

$f_y(x,y) = 4x - 4y = 0 \Rightarrow x = y \Rightarrow -3y^2 + 4y = 0$

$\Rightarrow -y(3y-4) = 0$

$y = 0, \quad y = \frac{4}{3}$

$x = 0, \quad x = \frac{4}{3}$

CRITICAL POINTS $(0,0)$ & $\left(\frac{4}{3}, \frac{4}{3}\right)$

$$d(x,y) = \begin{vmatrix} -6x & 4 \\ 4 & -4 \end{vmatrix} = 24x - 16$$

$d(0,0) = -16 < 0 \Rightarrow$ $\boxed{(0,0,1) \text{ IS A SADDLE POINT.}}$

$d\left(\frac{4}{3}, \frac{4}{3}\right) = 16 > 0$ & $f_{xx}\left(\frac{4}{3}, \frac{4}{3}\right) = -8 < 0$

$\Rightarrow$ $\boxed{f\left(\frac{4}{3}, \frac{4}{3}\right) = \frac{59}{27} \text{ IS A RELATIVE MAX.}}$

2. (2 points) It is easy to see that any point of the form $(0, b)$ or $(a, 0)$ is a critical point of $f(x, y) = x^2y^2$. However, the second derivative test is inconclusive at each of these points since they give rise to zero determinants. Nonetheless, each point yields a minimum value. How can we be so sure of this?

BECAUSE $f(x,y) = x^2y^2$ IS A PRODUCT OF SQUARES IT MUST ALWAYS BE NON NEGATIVE.

MIN VALUE MUST BE $f(0,b) = f(a,0) = 0$.

3. (3.5 points) Use Lagrange multipliers to find the extreme values of $f(x, y) = x^2y$ subject to the constraint $x^2 + y^2 = 1$.

$$\vec{\nabla} f(x,y) = 2xy\,\hat{i} + x^2\,\hat{j}$$

$$g(x,y) = x^2 + y^2 \Rightarrow \vec{\nabla} g(x,y) = 2x\,\hat{i} + 2y\,\hat{j}$$

$$2xy = \lambda 2x \Rightarrow 2xy - 2\lambda x = 0$$

$$x^2 = \lambda 2y \qquad 2x(y-\lambda) = 0$$

$$x^2 + y^2 = 1 \qquad x = 0,\ y = \lambda$$

$$\Downarrow \qquad\qquad \Downarrow$$

$$y = \pm 1 \qquad x^2 = 2y^2$$

$$3y^2 = 1$$

$$y = \pm\frac{1}{\sqrt{3}} \Rightarrow x = \pm\sqrt{\frac{2}{3}}$$

$$f\left(\pm\sqrt{\frac{2}{3}}, \sqrt{\frac{1}{3}}\right) = \sqrt{\frac{4}{27}}$$

IS THE MAX.

$$f\left(\pm\sqrt{\frac{2}{3}}, -\sqrt{\frac{1}{3}}\right) = -\sqrt{\frac{4}{27}}$$

IS THE MIN.

CRIT POINTS ARE

$$(0,1),\ (0,-1),\ \left(\sqrt{\frac{2}{3}}, \sqrt{\frac{1}{3}}\right),$$

$$\left(-\sqrt{\frac{2}{3}}, \sqrt{\frac{1}{3}}\right),\ \left(\sqrt{\frac{2}{3}}, -\sqrt{\frac{1}{3}}\right),\ \left(-\sqrt{\frac{2}{3}}, -\sqrt{\frac{1}{3}}\right)$$