

# Math 173 - Test 1

February 17, 2010

Name key

Score \_\_\_\_\_

Show all work. Supply explanations when necessary.

1. (6 points) The vectors  $\vec{v}$  and  $\vec{w}$  are 2D-vectors in the  $xy$ -plane.  $\vec{v}$  has magnitude 50 and makes an angle of  $210^\circ$  with the positive  $x$ -axis.  $\vec{w}$  has magnitude 30 and makes an angle of  $45^\circ$  with the positive  $x$ -axis. Find  $\vec{v} + \vec{w}$  and write your answer in terms of  $\hat{i}$  and  $\hat{j}$ .

$$\vec{v} = 50 \cos 210^\circ \hat{i} + 50 \sin 210^\circ \hat{j} = -25\sqrt{3} \hat{i} - 25 \hat{j}$$

$$\vec{w} = 30 \cos 45^\circ \hat{i} + 30 \sin 45^\circ \hat{j} = 15\sqrt{2} \hat{i} + 15\sqrt{2} \hat{j}$$

$$\boxed{\vec{v} + \vec{w} = -22.088 \hat{i} - 3.787 \hat{j}}$$

2. (6 points) Let  $P$  and  $Q$  be the points  $(3, 5, -2)$  and  $(5, 3, -6)$ , respectively.

- (a) Find the midpoint of the line segment connecting  $P$  and  $Q$ .

$$\left( \frac{5+3}{2}, \frac{3+5}{2}, \frac{-6+(-2)}{2} \right)$$

$$= \boxed{(4, 4, -4)}$$

- (b) Find a vector of magnitude 7 that is parallel to the line segment connecting  $P$  and  $Q$ .

$$\vec{PQ} = 2\hat{i} - 2\hat{j} - 4\hat{k}$$

$$|\vec{PQ}| = \sqrt{4+4+16} = \sqrt{24} = 2\sqrt{6}$$

$$\boxed{\frac{7}{2\sqrt{6}} (2\hat{i} - 2\hat{j} - 4\hat{k})}$$

3. (8 points) Find the angle between the planes described by the equations

$$2x + 3y + 5z = 12 \text{ and } 3x - y + 2z = 9.$$

$$\vec{n}_1 = 2\hat{i} + 3\hat{j} + 5\hat{k} \quad \vec{n}_2 = 3\hat{i} - \hat{j} + 2\hat{k}$$

$$\cos \theta = \frac{\vec{n}_1 \cdot \vec{n}_2}{|\vec{n}_1| |\vec{n}_2|} = \frac{6 - 3 + 10}{\sqrt{38} \sqrt{14}}$$

$$\theta = \cos^{-1} \left( \frac{13}{\sqrt{38} \sqrt{14}} \right) \approx 55.69^\circ$$

4. (12 points) Consider the triangle with vertices  $A(4, 2, -1)$ ,  $B(3, 3, 3)$ , and  $C(-7, -1, 2)$ .

- (a) Compute  $\vec{AB} \times \vec{AC}$ .

$$\vec{AB} = -\hat{i} + \hat{j} + 4\hat{k}$$

$$\vec{AC} = -11\hat{i} - 3\hat{j} + 3\hat{k}$$

$$\vec{AB} \times \vec{AC} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -1 & 1 & 4 \\ -11 & -3 & 3 \end{vmatrix}$$

$$= 15\hat{i} - 41\hat{j} + 14\hat{k}$$

- (b) Find the area of  $\triangle ABC$ .

$$\text{Area} = \frac{1}{2} |\vec{AB} \times \vec{AC}|$$

$$= \frac{1}{2} \sqrt{225 + 1681 + 196} = \frac{1}{2} \sqrt{2102} \approx 22.92$$

- (c) Find an equation of the plane containing  $\triangle ABC$ .

Using  $(3, 3, 3)$ :

$$15(x-3) - 41(y-3) + 14(z-3) = 0$$

$$15x - 41y + 14z = -36$$

5. (8 points) Suppose  $\frac{d\vec{r}}{dt} = 6e^{3t}\hat{i} + (\cos t)\hat{j} + \ln(t+1)\hat{k}$ . Find  $\vec{r}(t)$  if  $\vec{r}(0) = 3\hat{i} - 2\hat{j} + 5\hat{k}$ .

$$\begin{aligned}\vec{r}(t) &= \int (6e^{3t}, \cos t, \ln(t+1)) dt \\ &= (2e^{3t} + c_1)\hat{i} + (c_2 + \sin t)\hat{j} + (c_3 + (t+1)\ln(t+1) - (t+1))\hat{k}\end{aligned}$$

$$\vec{r}(0) = 3\hat{i} - 2\hat{j} + 5\hat{k} = (2+c_1)\hat{i} + c_2\hat{j} + (c_3-1)\hat{k} \Rightarrow c_1 = 1, c_2 = -2, c_3 = 6$$

$$\boxed{\vec{r}(t) = (2e^{3t} + 1)\hat{i} + (-2 + \sin t)\hat{j} + ((t+1)\ln(t+1) - t + 5)\hat{k}}$$

6. (10 points) Consider the surface described by the equation  $z^2 = x^2 + \frac{y^2}{4}$ .

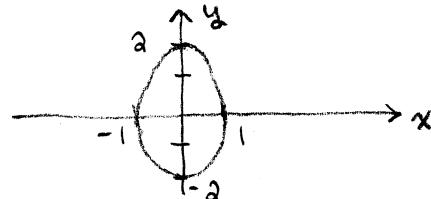
(a) Identify the surface.

Cone

- (b) Describe (or sketch) in detail the level curve obtained by fixing  $z = 1$ .

$$x^2 + \frac{y^2}{4} = 1$$

DESCRIBES  
AN ELLIPSE



- (c) Describe (or sketch) in detail the level curve(s) obtained by fixing  $y = 0$ .

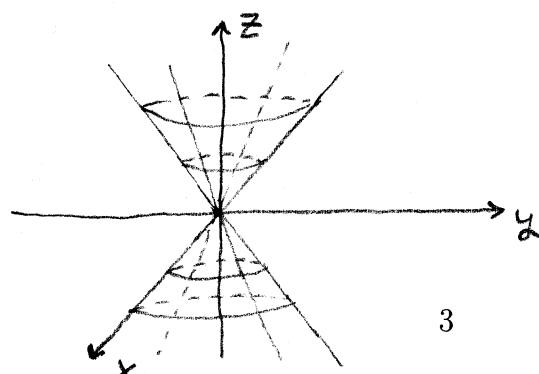
$$z^2 = x^2$$

or  $z = \pm x$

THE  $45^\circ / -45^\circ$  LINES

IN THE  $XZ$ -PLANE

- (d) Sketch a rough graph of the surface.



7. (4 points) Find the domain of the vector-valued function  $\vec{f}(t) = \sqrt{t}\hat{i} + (\sin t)\hat{j} - \frac{1}{\sin t}\hat{k}$ .

$$\sqrt{t} \quad t \geq 0$$

$\sin t$  — All real #'s

$\frac{1}{\sin t}$  — CAN'T HAVE  $\sin t = 0$   
i.e.  $t \neq n\pi$

DOMAIN:

$\{ t : t \geq 0 \text{ AND } t \text{ IS NOT A MULTIPLE OF } \pi \}$

8. (9 points) A projectile is launched at a height of 10 feet above the ground with an initial velocity of 88 feet per second and at an angle of  $30^\circ$  above the horizontal.

- (a) Find the vector-valued function that gives the projectile's position at any time  $t$ .

$$\vec{r}(t) = 88 \cos 30^\circ t \hat{i} + [-16t^2 + 88 \sin 30^\circ t + 10] \hat{j}$$

$$\boxed{\vec{r}(t) = 44\sqrt{3}t \hat{i} + [-16t^2 + 44t + 10] \hat{j}}$$

- (b) What is the maximum height of the projectile?

$$\text{Set } \vec{r}_y'(t) = 0 \dots$$

$$-32t + 44 = 0$$

$$\Rightarrow t = \frac{44}{32} = 1.375 \text{ s}$$

Height at  $t = 1.375$ :

$$-16(1.375)^2 + 44(1.375) + 10$$

$$= \boxed{40.25 \text{ FT}}$$

- (c) What is the projectile's height when it has covered a horizontal distance of 150 ft?

$$\text{Find } \vec{r}_y(t) \text{ when } \vec{r}_x(t) = 150 \dots$$

$$44\sqrt{3}t = 150$$

$$\Rightarrow t = \frac{150}{44\sqrt{3}} \approx 1.968 \text{ s}$$

Height at  $t = 1.968 \text{ s}$ :

$$\approx \boxed{34.619 \text{ FT}}$$

9. (8 points) A particle is moving along the helix described by the vector-valued function  $\vec{r}(t) = (2 \sin t)\hat{i} + (2 \cos t)\hat{j} + 2t\hat{k}$ . Find the particle's unit tangent vector.

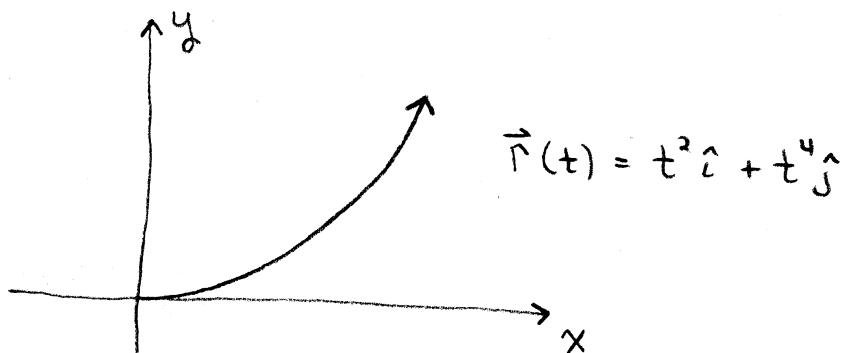
$$\vec{r}'(t) = 2 \cos t \hat{i} - 2 \sin t \hat{j} + 2\hat{k}$$

$$|\vec{r}'(t)| = \sqrt{4 \cos^2 t + 4 \sin^2 t + 4} = \sqrt{8} = 2\sqrt{2}$$

$$\hat{T}(t) = \frac{\vec{r}'(t)}{|\vec{r}'(t)|} \Rightarrow \boxed{\hat{T}(t) = \frac{1}{\sqrt{2}} (\cos t \hat{i} - \sin t \hat{j} + \hat{k})}$$

10. (5 points) Sketch the graph of the vector-valued function  $\vec{r}(t) = t^2\hat{i} + t^4\hat{j}$ .

$$\begin{aligned} x &= t^2 \\ y &= t^4 \Rightarrow y = x^2 \text{ BUT MUST HAVE} \\ &\quad x \geq 0 \end{aligned}$$



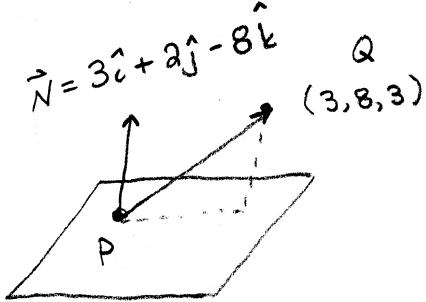
11. (6 points) Find a set of symmetric equations for the line passing through the points  $(3, -5, -3)$  and  $(7, 3, -2)$ .

$$\underbrace{\vec{v} = 4\hat{i} + 8\hat{j} + \hat{k}}$$

Using  $(3, -5, -3)$

$$\boxed{\frac{x-3}{4} = \frac{y+5}{8} = z+3}$$

12. (8 points) Find the distance from the plane  $3x + 2y - 8z = 10$  to the point  $(3, 8, 3)$ .



$$\vec{PQ} = 3\hat{i} + 7\hat{j} + 4\hat{k}$$

$$\text{DISTANCE} = \frac{|\vec{PQ} \cdot \vec{N}|}{|\vec{N}|}$$

P = POINT IN PLANE.

I'll use

$$P(0, 1, -1)$$

13. (6 points) Let  $\vec{x} = -3\hat{i} - \hat{j} + 4\hat{k}$  and  $\vec{y} = 5\hat{i} - 7\hat{k}$ . Find the projection of  $\vec{y}$  onto  $\vec{x}$ .

$$\begin{aligned} \text{Proj}_{\vec{x}} \vec{y} &= \frac{\vec{x} \cdot \vec{y}}{\vec{x} \cdot \vec{x}} \vec{x} \\ &= \frac{-15 + 0 - 28}{9 + 1 + 16} (-3\hat{i} - \hat{j} + 4\hat{k}) \end{aligned}$$

$$= \boxed{\frac{-43}{26} (-3\hat{i} - \hat{j} + 4\hat{k})}$$

14. (4 points) Sketch or describe the surface in space defined by the equation  $y = x^2 + 1$ .

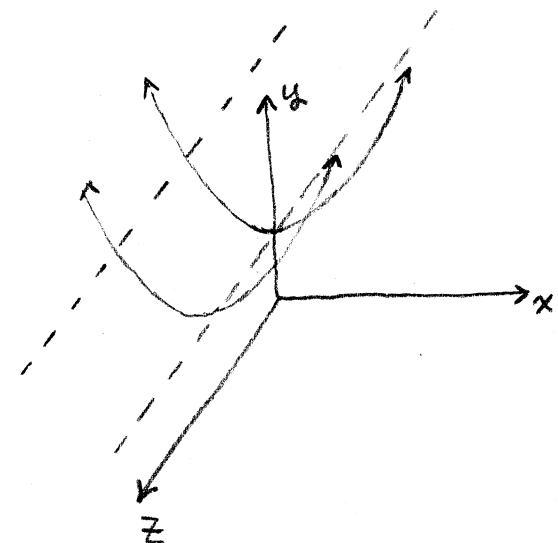
THE SURFACE IS A PARABOLIC

CYLINDER WHOSE GENERATING

CURVE IS THE PARABOLA

IN THE XY-PLANE,  $y = x^2 + 1$ ,

WHOSE GENERATING  
LINE IS THE Z-AXIS.



15. (4 points extra credit) To open the 1992 Summer Olympics in Barcelona, bronze medalist archer Antonio Rebollo lit the Olympic torch with a flaming arrow. Suppose that Rebollo shot the arrow at a height of 6 ft above the ground 90 ft from a 70-ft-high cauldron, and he wanted the arrow to reach its maximum height exactly 4 ft above the center of the cauldron. Find the initial speed and firing angle of the arrow.

$$\vec{r}(t) = v_0 \cos \theta t \hat{i} + [ -16t^2 + v_0 \sin \theta t + 6 ] \hat{j}$$

MUST SIMULTANEOUSLY SOLVE

$$-32t + v_0 \sin \theta = 0 \rightarrow v_0 \sin \theta = 32t$$

$$v_0 \cos \theta t = 90$$

$$-16t^2 + v_0 \sin \theta t + 6 = 74$$

$$-16t^2 + 32t + 6 = 74$$

$$16t^2 = 68$$

$$t^2 = \frac{68}{16} \Rightarrow t = 2.06155\dots$$

$$v_0 \sin \theta = 32t \quad \Rightarrow \quad \tan \theta = \frac{32}{90} t^2 = \frac{32}{90} \cdot \frac{68}{16} = \frac{68}{45}$$

$$v_0 \cos \theta = \frac{90}{t}$$

$$\theta = \tan^{-1}\left(\frac{68}{45}\right) \approx 56.505^\circ$$

$$v_0 = \frac{90}{t \cos \theta} \approx 79.107 \text{ FT/s}$$