Math 173 - Quiz 7

March 8, 2012

Name key

Score _____

Show all work to receive full credit. Supply explanations when necessary.

1. (3 points) Use the ϵ_1-ϵ_2 definition of differentiable to show that $f(x,y) = 2x^2 - 5xy + 4y^2$ is differentiable everywhere.

NOTICE THAT $f_x(x,y) = 4x - 5y$ AND $f_y(x,y) = -5x + 8y$.

NOW WE HAVE

$$\begin{aligned}
\Delta z &= f(x+\Delta x, y+\Delta y) - f(x,y) \\
&= \left(2x^2 + 4x\Delta x + 2\Delta x^2 - 5xy - 5x\Delta y - 5y\Delta x - 5\Delta x\Delta y + 4y^2 + 8y\Delta y + 4\Delta y^2\right) \\
&\quad - \left(2x^2 - 5xy + 4y^2\right) \\
&= 4x\Delta x + 2\Delta x^2 - 5x\Delta y - 5y\Delta x - 5\Delta x\Delta y + 8y\Delta y + 4\Delta y^2 \\
&= (4x - 5y)\Delta x + (-5x + 8y) + (2\Delta x)\Delta x + (-5\Delta x + 4\Delta y)\Delta y \\
&= f_x\,\Delta x + f_y\,\Delta y + \epsilon_1\Delta x + \epsilon_2\Delta y
\end{aligned}$$

SINCE $\epsilon_1 = \Delta x$ AND $\epsilon_2 = (-5\Delta x + 4y)$ BOTH APPROACH ZERO AS $(\Delta x, \Delta y) \to (0,0)$, f IS DIFFERENTIABLE WHEREVER THE EQUALITY HOLDS, I.E. EVERYWHERE.

2. (2 points) Assume that y is implicitly defined as a function of x and z by the following equation:

$$x \ln y + y^2 z + z^2 = 8.$$

Find $\dfrac{\partial y}{\partial x}$ and $\dfrac{\partial y}{\partial z}$.

LET $F(x,y,z) = x\ln y + y^2 z + z^2$

$$\frac{\partial y}{\partial x} = -\frac{F_x}{F_y} = \frac{-\ln y}{\frac{x}{y} + 2yz} = \boxed{\frac{-y\ln y}{x + 2y^2 z}}$$

$$\frac{\partial y}{\partial z} = -\frac{F_z}{F_y} = \frac{-(y^2 + 2z)}{\frac{x}{y} + 2yz} = \boxed{\frac{-(y^3 + 2yz)}{x + 2y^2 z}}$$

3. (3 points) Page 923, Problem #30

$$r = \sqrt{x^2+y^2} \Rightarrow dr = \frac{1}{2}(x^2+y^2)^{-1/2}(2x\,dx + 2y\,dy)$$

$$\theta = \tan^{-1}\left(\frac{y}{x}\right) \Rightarrow d\theta = \frac{1}{\left(\frac{y}{x}\right)^2+1}\left(-\frac{y}{x^2}dx + \frac{1}{x}dy\right)$$

$$\Delta r \approx \left(7.2^2+2.5^2\right)^{-1/2}\left(7.2(0.05)+2.5(0.05)\right) \approx \boxed{0.064} \quad \Delta r$$

$$\Delta\theta \approx \frac{1}{\left(\frac{2.5}{7.2}\right)^2+1}\left(-\frac{2.5}{7.2^2}(0.05)+\frac{1}{7.2}(0.05)\right) \approx \boxed{0.004} \quad \Delta\theta$$

4. (2 points) Page 944, Problem #66 $h(x,y) = 5000 - 0.001x^2 - 0.004y^2$

$$\vec{\nabla} h(x,y) = -0.002x\,\hat{i} - 0.008y\,\hat{j}$$

$$\boxed{\vec{\nabla} h(500,300) = -\hat{i} - 2.4\hat{j}}$$