Math 173 - Test 2a

March 15, 2012

Name ______________________

Score __________

Show all work to receive full credit. Supply explanations where necessary.

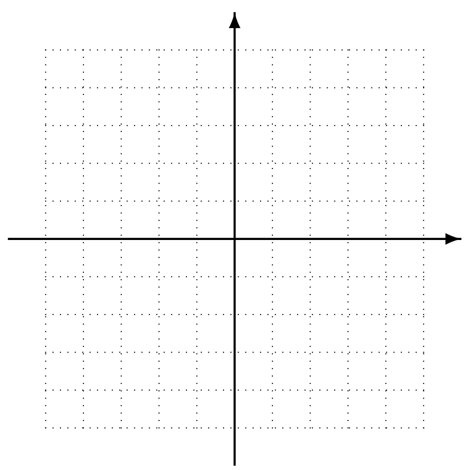
1. (10 points) Determine the limit or show that it does not exist.

   (a) $\displaystyle \lim_{(x,y)\to(0,0)} \frac{xy - x}{x^2y^2 + x}$

   (b) $\displaystyle \lim_{(x,y)\to(2,2)} \frac{y - 2}{x^2 - 4}$

2. (10 points) Consider the graph of the function $f(x, y) = 2x^2 - 4xy^2$. Find parametric equations for the normal line and an equation for the tangent plane at the point $(-1, 2)$.

3. (5 points) Describe the domain of $f(x, y) = \ln(4 - x - y)$. Then sketch the level curve $f(x, y) = 1$.



4. (5 points) Suppose z is a function of x, y and x, y are functions of t, u, v. Write the chain rule formulas for $\dfrac{\partial z}{\partial t}$ and $\dfrac{\partial z}{\partial u}$.

5. (8 points) A bug located at $(3, 9, 4)$ begins walking in a straight line toward $(5, 7, 3)$. At what rate is the bug's temperature changing at the initial point if the temperature is described by $T(x, y, z) = xe^{y-z}$.

6. (6 points) Let $h(x, y) = \ln(x^3 + y^3)$. Find $h_{xy}(x, y)$. Without computing h_{yx}, what can you say about this other mixed partial derivative? Briefly explain.

7. (4 points) Suppose f is a function such that $f''(x)$ exits for all x. Can the curvature function ever be greater than it is at a relative extreme point. Explain.

8. (4 points) Two cars are racing around a circular track. At a certain moment, both speedometers read 120 mph. Which component of acceleration, tangential or normal, is the same for each car at that moment? Explain. (Hint: Refer to the formulas on your laminated sheet.)

9. (8 points) Find and classify the critical points of $g(x, y) = x^2 - xy + 2y^2 + 7x$.

Math 173 - Test 2b

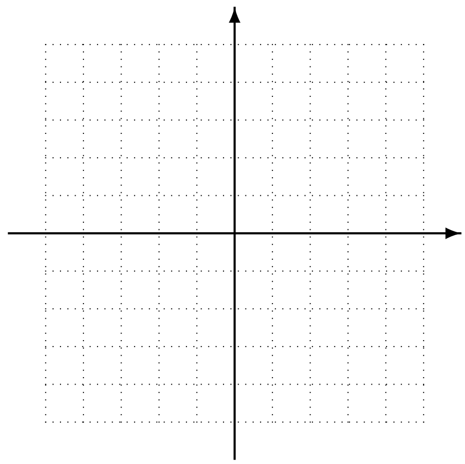
March 15, 2012

Name ____________________

Score ____________

Show all work to receive full credit. Supply explanations where necessary.

1. (8 points) Calculate the curvature $\kappa(t)$ of the twisted cubic $\vec{r}(t) = t\hat{\imath} + t^2\hat{\jmath} + t^3\hat{k}$. Then sketch the graph of $\kappa(t)$ (You may use your graphing calculator or a CAS.) and use the graph to determine the point at which the curvature is greatest.



2. (10 points) The linearization of a function $F(x, y)$ at the point (x_0, y_0) is the function

$$L(x, y) = F(x_0, y_0) + F_x(x_0, y_0)(x - x_0) + F_y(x_0, y_0)(y - y_0).$$

Suppose we are given the two functions

$$f(x, y) = 4x^2 + y^2 - 4, \qquad g(x, y) = x + y - \sin(x - y)$$

and we wish to solve the system of nonlinear equations $f(x, y) = g(x, y) = 0$. We can approximate a solution using linearizations and Newton's method.

(a) Let $(x_0, y_0) = (1, 0)$ be our initial guess at the solution. Let $f_0(x, y)$ and $g_0(x, y)$ be the linearizations for f and g at the point (x_0, y_0). Find $f_0(x, y)$ and $g_0(x, y)$.

(b) Solve the linear system of equations

$$f_0(x, y) = 0, \qquad g_0(x, y) = 0.$$

(c) Let (x_1, y_1) be the solution of the linear system in part (b). It represents an improved guess at the solution. Compute $f(x_1, y_1)$ and $g(x_1, y_1)$.

(d) (Extra Credit 4 pts) On a separate sheet of paper, use (x_1, y_1) in place of (x_0, y_0) and repeat the steps above to further improve our guess at the solution.

3. (10 points) Find and classify the critical points of $f(x, y) = y^2x - yx^2 + xy$.

4. (8 points) Suppose that $y = \left(\dfrac{2L^2v_0^2}{3g}\right)^{1/3}$. Use differentials to estimate the progagated error, Δy, if the measured values of L, v_0, and g are 1.546, 0.84, and 9.78, respectively, and the measurement errors are $\Delta L = 0.005$, $\Delta v_0 = 0.1$, and $\Delta g = 0.003$.

5. (4 points) If an object's speed is constant, what can be said about its tangential component of acceleration? Use a formula for a_T (see page 877) to argue that, in this case, $\vec{v}$ and $\vec{a}$ are orthogonal.