Math 173 - Quiz 3

February 14, 2013

Name key

Score _____

Show all work to receive full credit. Supply explanations when necessary.

1. (3 points) Make a small table of values (3 points or so) for the vector-valued function $\vec{r}(t) = 3t^2\hat{\imath} - t^3\hat{\jmath}$. Then plot your points and sketch a rough graph of $\vec{r}(t)$.

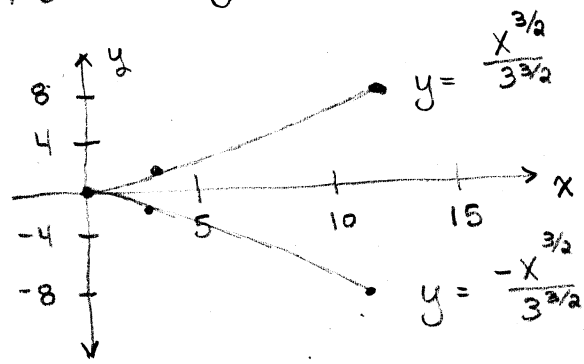
t	x	y
0	0	0
1	3	-1
-1	3	1
2	12	-8
-2	12	8

$x = 3t^2 \Rightarrow t = \pm\sqrt{\frac{x}{3}}$

$y = -t^3$

$\Downarrow$

$y = \mp\sqrt{\frac{x}{3}}^{\,3} \Rightarrow y = \mp \frac{x^{3/2}}{3^{3/2}}$



2. (3 points) Find $\vec{r}(t)$ if $\vec{r}'(t) = e^{2t}\hat{\imath} + \sec^2(t)\hat{\jmath} + (t^2+3)\hat{k}$ and $\vec{r}(0) = 3\hat{\imath} + \hat{\jmath} - 4\hat{k}$.

$\vec{r}(t) = \left(\frac{1}{2}e^{2t} + c_1\right)\hat{\imath} + \left(\tan t + c_2\right)\hat{\jmath} + \left(\frac{1}{3}t^3 + 3t + c_3\right)\hat{k}$

$\vec{r}(0) = 3\hat{\imath} + \hat{\jmath} - 4\hat{k} \Rightarrow c_1 = \frac{5}{2},\ c_2 = 1,\ c_3 = -4$

$$\boxed{\vec{r}(t) = \left(\frac{1}{2}e^{2t} + \frac{5}{2}\right)\hat{\imath} + \left(1 + \tan t\right)\hat{\jmath} + \left(\frac{1}{3}t^3 + 3t - 4\right)\hat{k}}$$

3. (4 points) An object is launched from the ground at an angle of 60°. Find the initial speed required so that the object will reach its maximum height precisely 100 ft downrange. $g = 32$

$\vec{r}(t) = \left(V_0 \cos 60^\circ t\right)\hat{\imath} + \left(-\frac{1}{2}(32)t^2 + V_0 \sin 60^\circ t\right)\hat{\jmath} = \frac{1}{2}V_0 t\,\hat{\imath} + \left(-16t^2 + \frac{\sqrt{3}V_0}{2}t\right)\hat{\jmath}$

$\frac{1}{2}V_0 t = 100$

$-32t + \frac{\sqrt{3}V_0}{2} = 0 \Rightarrow t = \frac{\sqrt{3}V_0}{64}$

$\Rightarrow \frac{\sqrt{3}}{128}V_0^2 = 100$

$V_0^2 = \frac{12800}{\sqrt{3}} \Rightarrow V_0 = \sqrt{\frac{12800}{\sqrt{3}}}$

$\approx \boxed{85.97 \text{ FT/S}}$