Math 173 - Test 2a

March 21, 2013

Name ____________________

Score __________

Show all work to receive full credit. Supply explanations where necessary.

1. (8 points) Find a set of parametric equations for the line normal to the surface at the point $(-1, 1, -1)$.

$$x^2y^3 - y^2z + 2xz^3 = 4$$

2. (6 points) Show that the limit does not exist: $\displaystyle\lim_{(x,y)\to(2,1)} \frac{2y^2 - x}{x^2 - 4y}$

3. (8 points) Use differentials to approximate Δz as (x, y) changes from $(1, 1)$ to $(1.01, 0.98)$.

$$f(x, y) = \sqrt{5xy + 4x^2y^4}$$

4. (5 points) Suppose $w = f(x, y)$ is a differentiable function of x and y. Let $x = u - v$ and $y = v - u$. Use the chain rule to determine $\dfrac{\partial w}{\partial u} + \dfrac{\partial w}{\partial v}$.

5. (6 points) Assume that the following equation implicitly defines z as a function of x and y. Find $\partial z/\partial x$ at the point $(\frac{\pi}{4}, 0, \pi)$.

$$\tan(x + y) + \sin(y + z) = 1$$

6. (10 points) Find and classify the critical points of $f(x, y) = 2x^2 + 2xy + y^2 - 2x - 3$.

7. (10 points) Consider the function $g(x, y) = \tan^{-1}(\frac{y}{x})$.

   (a) Find an equation of the plane tangent to the graph of g at the point $(1, 1, \frac{\pi}{4})$.

   (b) Find the linearization of g at $(1, 1)$.

   (c) Use your linearization to approximate $g(1.05, 0.97)$.

8. (4 points) Suppose $z = 2xe^{5y} - 3ye^{-x}$. Find $\dfrac{\partial^2 z}{\partial x \partial y}$.

9. (10 points) Consider the surface described by the equation $4x^2 - 3y^2 + 12z^2 + 3 = 0$.

(a) Describe the level curve at $y = 2$.

(b) Is there a level curve at $y = 0$? Explain.

(c) Describe the level curve $z = 1$.

(d) What are the level curves in planes parallel to the yz-plane?

(e) Identify the quadric surface and roughly sketch the graph.

10. (5 points) Determine the limit: $\displaystyle\lim_{(x,y)\to(0,0)} \frac{x^4 - 4y^4}{x^2 + 2y^2}$

11. (8 points) Consider the function $f(x, y, z) = xy^3 - \dfrac{\sqrt{x}}{z^2}$.

(a) Find the directional derivative of f at $(4, 1, 2)$ in the direction of $\hat{\imath} + 2\hat{\jmath} - \hat{k}$.

(b) Find a unit vector in the direction of maximum increase of f.

Math 173 - Test 2b

March 21, 2013

Name ______________________

Score __________

Show all work to receive full credit. Supply explanations where necessary. This portion of the test is due Monday, April 1. You must work individually on this test.

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1. (6 points) Use the definition of differentiability to show that $f(x,y) = x + x^2y - 2y$ is differentiable everywhere.

2. (7 points) Find and classify the critical points of $g(x, y) = y^3 - 3yx^2 - 3y^2 - 3x^2 + 1$.

3. (4 points) The period T of a pendulum of length L is $T = 2\pi\sqrt{L/g}$, where g is the acceleration due to gravity. A pendulum is moved from a place where $g = 32.09\,\mathrm{ft/s^2}$ to a place where $g = 32.23\,\mathrm{ft/s^2}$. Also accompanying the change in g is a change in temperature that results in the length of the pendulum decreasing from $2.50\,\mathrm{ft}$ to $2.48\,\mathrm{ft}$. Use differentials to approximate the corresponding change in the period of the pendulum.

4. (3 points) Suppose $g(x,y) = \dfrac{4x^2y^2}{x^2+y^2}$ was to be defined at $(0,0)$ in such a way that it is continuous everywhere. What value must it be given at $(0,0)$?