

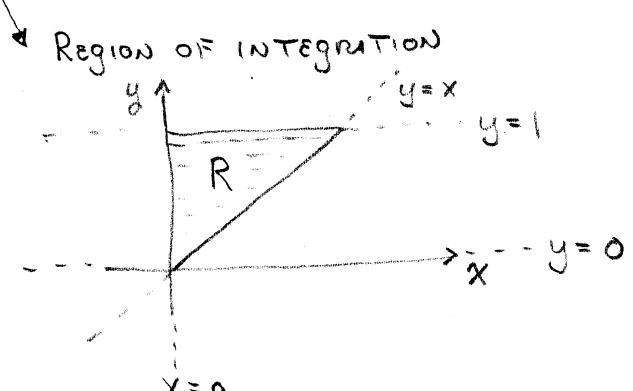
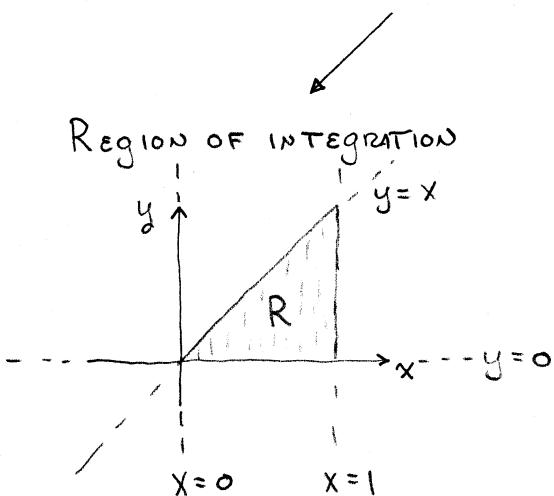
Show all work to receive full credit. Supply explanations where necessary.

1. (6 points) Evaluate the iterated integral by hand. Show all work, but you may use your calculator to check your work.

$$\begin{aligned}
 & \int_0^1 \int_0^x \int_0^{xy} x \, dz \, dy \, dx \\
 &= \int_0^1 \int_0^x xz \Big|_{z=0}^{z=xy} \, dy \, dx = \int_0^1 \int_0^x xy \, dy \, dx = \int_0^1 \frac{xy^2}{2} \Big|_{y=0}^{y=x} \, dx \\
 &= \int_0^1 \frac{x^4}{2} \, dx = \frac{x^5}{10} \Big|_0^1 = \boxed{\frac{1}{10}}
 \end{aligned}$$

2. (4 points) Do you expect this statement to be true? Explain.

$$\int_0^1 \int_0^x f(x, y) \, dy \, dx = \int_0^1 \int_0^y f(x, y) \, dx \, dy$$

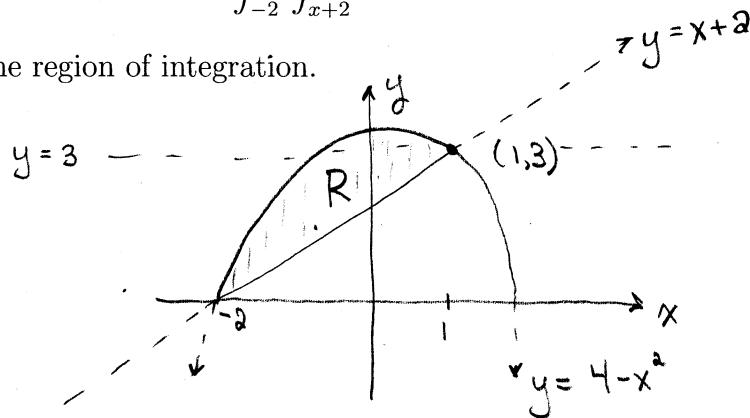


THE STATEMENT IS FALSE. THE REGIONS OF
 INTEGRATION ARE DIFFERENT. IN GENERAL,
 THE INTEGRALS WILL HAVE DIFFERENT VALUES.

3. (10 points) Consider the following iterated integral.

$$\int_{-2}^1 \int_{x+2}^{4-x^2} (x+y) dy dx$$

(a) Sketch the region of integration.



(b) Rewrite the integral with the order of integration reversed.

NEEDS
TWO
INTEGRALS.

$$\int_{y=0}^{y=3} \int_{x=-\sqrt{4-y}}^{x=y-2} (x+y) dx dy + \int_{y=3}^{y=4} \int_{x=-\sqrt{4-y}}^{x=\sqrt{4-y}} (x+y) dx dy$$

4. (10 points) Use the method of Lagrange multipliers to find the extreme values of $f(x, y) = x^2y$ subject to $x + 2y = 2$.

$$2xy = \lambda \Rightarrow 4xy = 2\lambda$$

$$x^2 = 2\lambda \quad 4xy = x^2$$

$$4xy - x^2 = 0$$

$$x(4y - x) = 0$$

$$x = 0$$

$$y = 1$$

$$(0, 1)$$

$$x = 4y$$

$$6y = 2$$

$$y = \frac{1}{3}$$

$$x = \frac{4}{3}$$

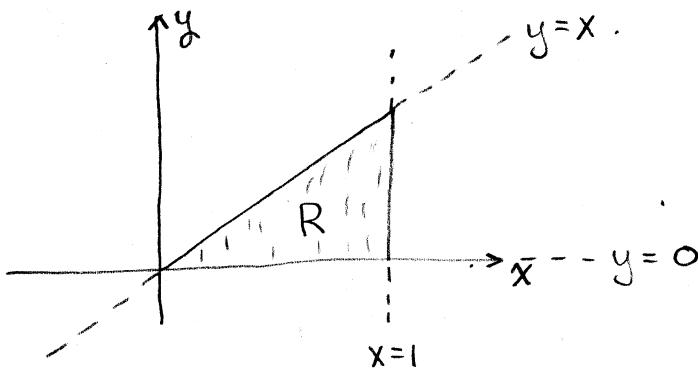
$$\left(\frac{4}{3}, \frac{1}{3}\right)$$

$$f(0, 1) = 0 = \text{MIN VALUE}$$

$$f\left(\frac{4}{3}, \frac{1}{3}\right) = \left(\frac{4}{3}\right)^2 \left(\frac{1}{3}\right) = \frac{16}{27}$$

= MAX VALUE

5. (10 points) A thin triangular plate is bounded by the graphs of $y = 0$, $y = x$, and $x = 1$. Find the mass of the plate if its density at (x, y) is given by $\rho(x, y) = \sqrt{1 - x^2}$. Evaluate the integral by hand, showing all work.



$$\text{MASS} = \int_{x=0}^{x=1} \int_{y=0}^{y=x} \sqrt{1-x^2} \, dy \, dx$$

$$= \int_0^1 x \sqrt{1-x^2} \, dx$$

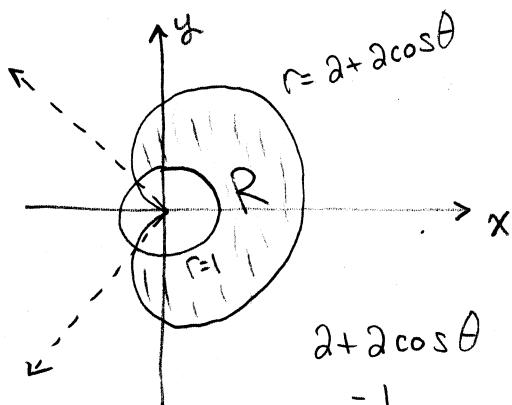
$$u = 1-x^2$$

$$du = -2x \, dx \quad -\frac{1}{2} du = x \, dx$$

$$\begin{aligned} -\frac{1}{2} \int_1^0 \sqrt{u} \, du &= \frac{1}{2} \int_0^1 u^{1/2} \, du \\ &= \frac{1}{2} \left(\frac{2}{3} \right) u^{3/2} \Big|_0^1 \end{aligned}$$

$$= \boxed{\frac{1}{3}}$$

6. (10 points) Let R be the polar region inside the graph of the $r = 2 + 2 \cos \theta$ and outside the graph of $r = 1$. Sketch the region R and then evaluate $\iint_R (2r + 3) dA$. You may use your calculator to evaluate the iterated integral.



$$\begin{aligned} r &= 2 + 2 \cos \theta \\ &= 1 \end{aligned}$$

$$\Rightarrow \cos \theta = -\frac{1}{2}$$

$$\theta = -\frac{2\pi}{3}, \frac{2\pi}{3}$$

$$\begin{array}{l} \theta = \frac{2\pi}{3} \quad r = 2 + 2 \cos \theta \\ \int \quad \int \quad (2r + 3) r dr d\theta \\ \theta = -\frac{2\pi}{3} \quad r = 1 \end{array}$$

$$\begin{aligned} &= \frac{484\pi + 53(3^{5/2})}{18} \\ &\approx 130.37 \end{aligned}$$

Test 3a - Problem 3

```
(%i1) integrate(integrate(x+y,y,x+2,4-x^2),x,-2,1);  
(%o1)  $\frac{171}{20}$   
  
(%i3) assume(y<4);  
(%o3) [y<4]  
  
(%i4) integrate(integrate(x+y,x,-sqrt(4-y),y-2),y,0,3)  
+ integrate(integrate(x+y,x,-sqrt(4-y),sqrt(4-y)),y,3,4);  
(%o4)  $\frac{171}{20}$ 
```

Test 3a - Problem 6

```
(%i6) 'integrate('integrate((2*r+3)*r,r,1,2+2*cos(theta)),theta,-2*%pi/3  
(%o6)  $\int_{-\frac{2\pi}{3}}^{\frac{2\pi}{3}} \int_1^{2\cos(\theta)+2} r(2r+3) dr d\theta$   
  
(%i7) ev(% , nouns);  
(%o7)  $\frac{484\pi + 533^{5/2}}{18}$   
  
(%i8) float(%);  
(%o8) 130.3732821971008
```

Math 173 - Test 3b

April 25, 2013

Name key Score _____

Show all work to receive full credit. Supply explanations where necessary. This portion of the test is due Monday, April 29. You must work individually on this test.

1. (9 points) Use the method of Lagrange multipliers to find the extreme values of $f(x, y) = xy$ subject to $\frac{x^2}{8} + \frac{y^2}{2} = 1$.

$$\begin{aligned} y &= \frac{\lambda x}{4} \\ x &= \lambda y \\ \frac{x^2}{8} + \frac{y^2}{2} &= 1 \end{aligned}$$

$$\left. \begin{array}{l} y = \frac{\lambda x}{4} \\ x = \lambda y \end{array} \right\} \quad x = \frac{\lambda^2 x}{4} \Rightarrow x - \frac{\lambda^2 x}{4} = 0$$

$$x \left(1 - \frac{\lambda^2}{4} \right) = 0$$

$$\begin{cases} x = 0 & \text{or} \\ \lambda = \pm 2 \end{cases}$$

$$\begin{array}{lll} \downarrow & \lambda = 2 & \lambda = -2 \\ y = 0 & x = 2y & x = -2y \\ \text{BUT } (0,0) & y^2 = 1 & y^2 = 1 \\ \text{DOES NOT SATISFY} & \downarrow & \downarrow \\ \text{THE CONSTRAINT} & y = \pm 1 & y = \pm 1 \\ (0,0) \text{ IS NOT A} & \downarrow & \downarrow \\ \text{CRITICAL} & x = \pm 2 & x = \mp 2 \\ \text{PT!} & & \\ & & (-2,1), (2,-1) \\ & & (2,1), (-2,-1) \end{array}$$

$$f(2,1) = 2$$

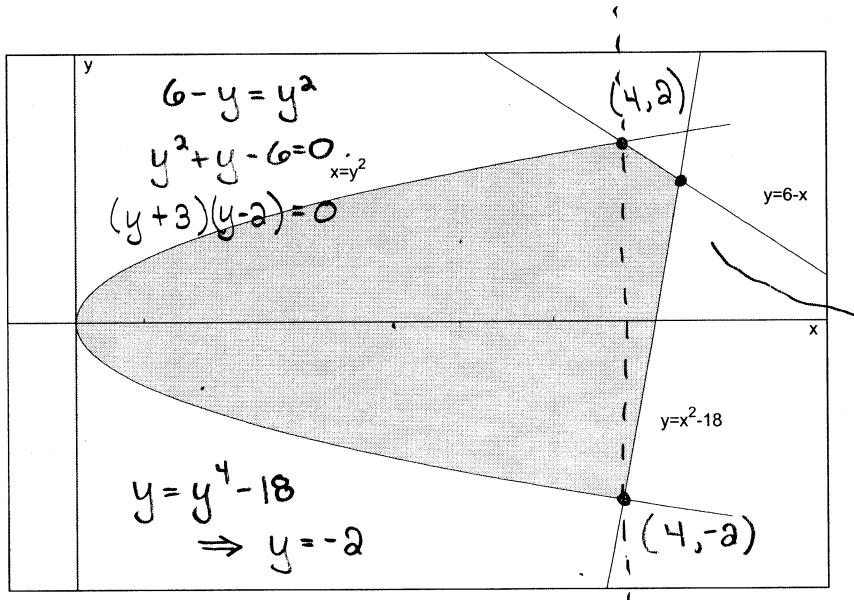
$$f(-2,-1) = 2$$

$$f(-2,1) = -2$$

$$f(2,-1) = -2$$

MAX VALUE IS 2
MIN VALUE IS -2

2. (9 points) The plane region R (shaded below) is bounded by the graphs of $x = y^2$, $y = 6 - x$, and $y = x^2 - 18$.



$$6-x = x^2 - 18$$

$$x^2 + x - 24 = 0$$

$$x = \frac{-1 \pm \sqrt{97}}{2}$$

$$x \approx 4.4244$$

$$y \approx 1.5756$$

Two integrals

REQUIRED WITH EITHER ORDER.

- (a) Evaluate $\iint_R dA$. After setting up your iterated integral(s), you may use your calculator for the evaluation.

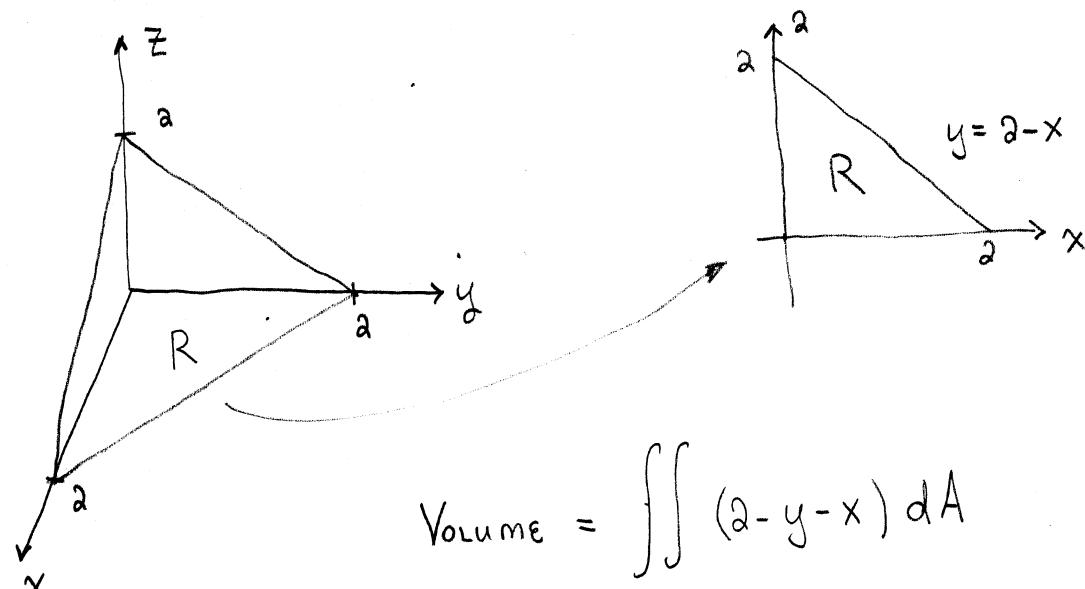
$$\iint_R dA = \int_{x=0}^{x=4} \int_{y=-\sqrt{x}}^{y=\sqrt{x}} dy dx + \int_{x=4}^{x=\frac{-1+\sqrt{97}}{2}} \int_{y=x^2-18}^{y=6-x} dy dx = \boxed{\frac{(97)^{3/2} - 817}{12} \approx 11.528}$$

SHOULD SAY $Z = X \sin X$

- (b) Set-up the expression that gives the average value of $\overbrace{y = x \sin x}$ over R .

$$\text{Avg Value} = \frac{12}{(97)^{3/2} - 817} \left[\int_0^4 \int_{-\sqrt{x}}^{\sqrt{x}} (x \sin x) dy dx + \int_4^{\frac{-1+\sqrt{97}}{2}} \int_{x^2-18}^{6-x} (x \sin x) dy dx \right]$$

3. (9 points) Use a double (or triple) integral to find the volume of the space region in the first octant under the plane $x + y + z = 2$. You may use your calculator for the integral evaluation.

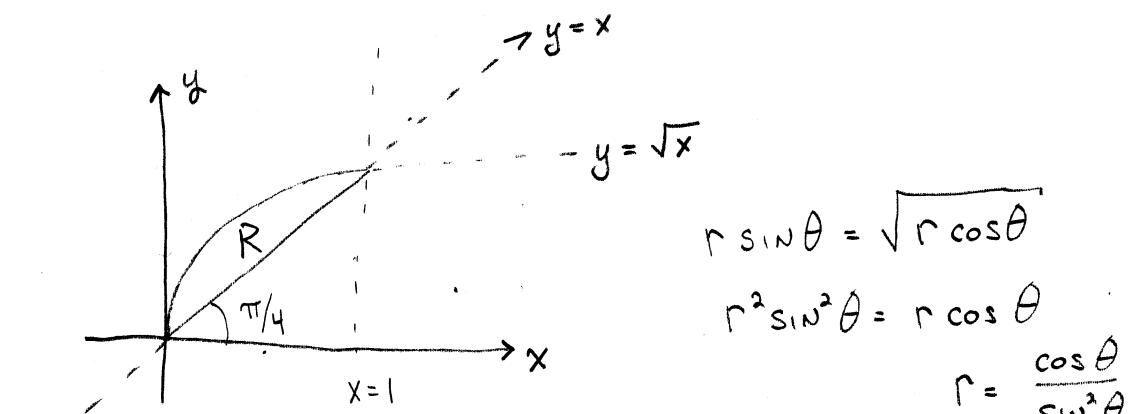


$$\text{Volume} = \iint_R (2 - y - x) dA$$

$$= \int_{x=0}^{x=2} \int_{y=0}^{y=2-x} (2 - y - x) dy dx$$

$$= \boxed{\frac{4}{3}}$$

4. (9 points) R is the first quadrant region between the graphs of $y = \sqrt{x}$ and $y = x$. Set-up the double integral, in both rectangular and polar coordinates, that gives the area of R .

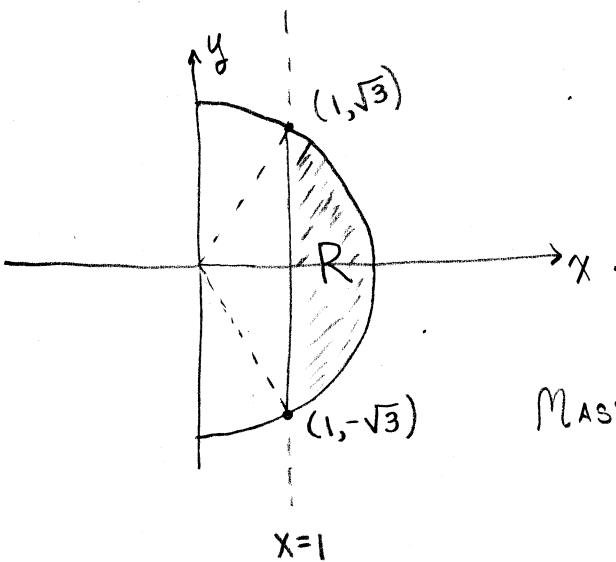


$$\text{Area of } R = \int_{x=0}^{x=1} \int_{y=x}^{y=\sqrt{x}} dy dx$$

$$\begin{aligned} & \theta = \frac{\pi}{2} \quad r = \frac{\cos \theta}{\sin^2 \theta} \\ & = \int_{\theta=0}^{\theta=\frac{\pi}{4}} \int_{r=0}^{r=\frac{\cos \theta}{\sin^2 \theta}} r dr d\theta \end{aligned}$$

$$= \frac{1}{6}$$

5. (9 points) A thin plate lies inside the circle $x^2 + y^2 = 4$ and to the right of $x = 1$. Find the center of mass of the plate if its density at the point (x, y) is given by $\rho(x, y) = x^2 + y^2 + 1$. You may use your calculator to evaluate any required integrals.



$$\begin{aligned} x &= 1 \\ r \cos \theta &= 1 \\ r &= \sec \theta \end{aligned}$$

$$\begin{aligned} \text{Mass} &= \iint \rho(x, y) dA \\ &= \int_{\theta = \tan^{-1}(\sqrt{3})}^{R} \int_{r=2}^{(r^2+1)r} (r^2+1) r dr d\theta \\ &\quad \theta = \tan^{-1}(-\sqrt{3}) \quad r = \sec \theta \\ &= 4\pi - 2\sqrt{3} \approx 9.10 \end{aligned}$$

$$\begin{aligned} M_y &= \int_{\theta = \tan^{-1}(-\sqrt{3})}^{\tan^{-1}(\sqrt{3})} \int_{r=\sec \theta}^2 r \cos \theta (r^2+1) r dr d\theta \\ &= \frac{38\sqrt{3}}{5} \approx 13.16 \end{aligned}$$

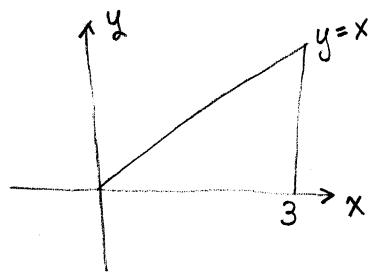
$$M_x = \int_{\theta = \tan^{-1}(-\sqrt{3})}^{\tan^{-1}(\sqrt{3})} \int_{r=\sec \theta}^2 r \sin \theta (r^2+1) r dr d\theta$$

$$\text{CENTER MASS} = \left(\frac{M_y}{M}, \frac{M_x}{M} \right) = \left(0, 0 \right) \quad \begin{matrix} \text{(obvious)} \\ \text{by symmetry} \end{matrix}$$

$$\left(\frac{M_y}{M}, \frac{M_x}{M} \right) \approx (1.45, 0)$$

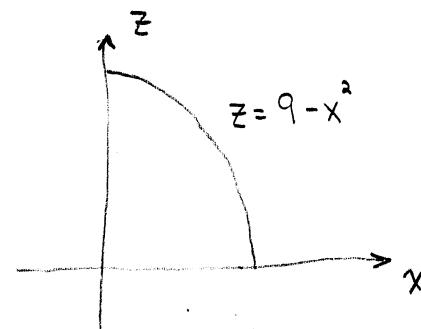
6. (5 points) Page 1036, Problem #38

$$\int_{x=0}^{x=3} \int_{y=0}^{y=x} \int_{z=0}^{z=9-x^2} dz dy dx = \frac{81}{4}$$



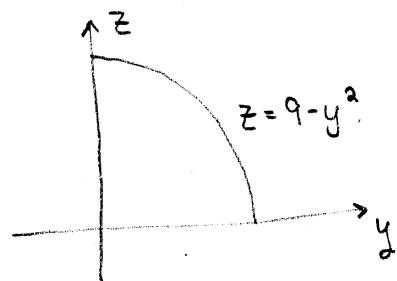
$$\int_{y=0}^{y=3} \int_{x=y}^{x=3} \int_{z=0}^{z=9-x^2} dz dx dy = \frac{81}{4}$$

$$\int_{x=0}^{x=3} \int_{z=0}^{z=9-x^2} \int_{y=0}^{y=x} dy dz dx = \frac{81}{4}$$



$$\int_{z=0}^{z=9} \int_{x=0}^{x=\sqrt{9-z}} \int_{y=0}^{y=x} dy dx dz = \frac{81}{4}$$

$$\int_{y=0}^{y=3} \int_{z=0}^{z=9-y^2} \int_{x=y}^{x=\sqrt{9-z}} dx dz dy = \frac{81}{4}$$



$$\int_{z=0}^{z=9} \int_{y=0}^{y=\sqrt{9-z}} \int_{x=y}^{x=\sqrt{9-z}} dx dy dz = \frac{81}{4}$$