

# Math 173 - Final Exam

May 13, 2013

Name \_\_\_\_\_

Score \_\_\_\_\_

Show all work. Supply explanations when necessary. **Unless otherwise specified, you may use your calculator or CAS to evaluate any integrals.** Each problem is worth 10 points.

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1. Find an equation of the plane passing through the three points  $P(2, 0, 3)$ ,  $Q(5, -2, 7)$ , and  $R(8, 3, -2)$ .

2. Find a set of parametric equations for the line passing through the point  $(7, 3, -1)$  and normal to the plane  $8x - 6y + 5z = 15$ .

3. Find the limit or show that it does not exist.

(a)  $\lim_{(x,y) \rightarrow (0,0)} \frac{xy^2}{x^2 + y^4}$

(b)  $\lim_{(x,y,z) \rightarrow (0,0,1)} \frac{zx + zy}{e^z(y^2 - x^2)}$

4. Let  $\vec{u}$  be the vector of magnitude 3 that has the direction of  $3\hat{i} + 12\hat{j} - 4\hat{k}$ . Let  $\vec{w}$  be the unit vector in the direction of  $2\hat{i} + 2\hat{j} + \hat{k}$ . Find the projection of  $\vec{w}$  onto  $\vec{u}$ .

5. The quarterback of a football team releases a pass at an angle of  $35^\circ$ , a speed of 54 ft/s, and a height of 7 ft above the playing field. A receiver 90 ft directly downfield prepares to catch the ball. At what height should he catch the ball? (Use  $g = 32 \text{ ft/s}^2$ .)

6. A surface is described by the equation  $xy^2 + 3x - z^2 = 8$ . Find an equation of the plane tangent to the surface at the point  $(1, -3, 2)$ .

7. Sketch the region of integration, reverse the order of integration, and evaluate the new iterated integral. You may use your calculator or CAS to evaluate the required integral.

$$\int_0^4 \int_{\sqrt{x}}^2 \frac{3}{2+y^2} dy dx$$

8. Use Lagrange multipliers to find the extreme values of  $f(x, y) = x^2y + 2y$  subject to  $x^2 + y^2 = 25$ .

9. Let  $\vec{r}(t) = 2t\hat{i} + t^2\hat{j} + 2t\hat{k}$ . Find the principal unit normal vector at  $t = 0$ . (Hint: Find  $\hat{T}'(0)$  and then normalize it.)

10. Show that  $\vec{F}(x, y) = \frac{1}{2}xy\hat{i} + \frac{1}{4}x^2\hat{j}$  is a conservative vector field. Then use any method to evaluate  $\int_C \vec{F} \cdot d\vec{r}$ , where  $C$  is any smooth curve from  $(0, 0)$  to  $(1, 1)$ . Evaluate your integral by hand.

11. The temperature distribution inside a containment vessel is given by

$$T(x, y, z) = \frac{4xe^{y^2}z^3}{x^2 + 1},$$

where the  $x, y, z$  are measured in centimeters and  $T$  is measured in degrees Celsius. Find a unit vector in the direction of maximum temperature increase at the point  $(1, 0, 1)$ .

12. Let  $\vec{r}(t) = 2\sqrt{t}\hat{i} + \frac{5\sin t}{t}\hat{j} + (t^2 + 2)\hat{k}$ .

(a) What is the domain of  $\vec{r}$ ?

(b) Find  $\lim_{t \rightarrow 0} \vec{r}(t)$ .

(c) What is the formal name we gave to the unit vector in the direction of  $d\vec{r}/dt$ ?

13. A right triangular prism in the 1st octant is bounded by the planes  $y = 0$ ,  $y = x$ ,  $x = 1$ ,  $z = 0$ , and  $z = 1$ . The density of the prism at the point  $(x, y, z)$  is given by  $\rho(x, y, z) = x^3 + y^2 + z + 1$ . Find the  $z$ -coordinate of the center of mass. Use your calculator or CAS to evaluate the required integrals.

14. Use cylindrical coordinates to find the volume of the solid bounded by the paraboloids  $z = 2 - x^2 - y^2$  and  $z = x^2 + y^2$ . Use your calculator or CAS to evaluate the required integral.

15. Use Green's Theorem to evaluate

$$\int_C \cos y \, dx + (xy - x \sin y) \, dy,$$

where  $C$  is the positively-oriented boundary of the region lying between the graphs of  $y = x$  and  $y = \sqrt{x}$ . Evaluate your integral by hand.