

Math 173 - Quiz 1

January 30, 2014

Name key

Score _____

Show all work to receive full credit. Supply explanations when necessary.

1. (3 points) The graph of the equation $(4-x)y^2 = x^3$ is called a *cissoid*. Find a unit vector normal to the cissoid at the point $(2, 2)$.

$$\frac{d}{dx}[(4-x)y^2] = 3x^2$$

$$2(4-x)\frac{dy}{dx} - y^2 = 3x^2$$

$$\frac{dy}{dx} = \frac{3x^2 + y^2}{2(4-x)y}$$

$$\left. \frac{dy}{dx} \right|_{(2,2)} = \frac{16}{8} = 2 \Rightarrow \begin{array}{l} \text{SLOPE OF} \\ \text{NORMAL VECTOR} \\ \text{IS } -\frac{1}{2} \end{array}$$

A NORMAL VECTOR IS $-2\hat{i} + \hat{j}$.

A UNIT NORMAL VECTOR IS $\frac{1}{\sqrt{5}}(-2\hat{i} + \hat{j})$.

2. (3 points) The vector \vec{u} has magnitude 4 and makes a 30° -angle with the positive x -axis. The vector \vec{w} has magnitude 4 and is parallel to $-2\hat{i} + \hat{j} + 3\hat{k}$. Find the component form of $\vec{u} + \vec{w}$.

$$\vec{u} = 4 \cos 30^\circ \hat{i} + 4 \sin 30^\circ \hat{j} = 2\sqrt{3}\hat{i} + 2\hat{j}$$

$$\vec{w} = \frac{4}{\sqrt{4+1+9}} (-2\hat{i} + \hat{j} + 3\hat{k}) = -\frac{8}{\sqrt{14}}\hat{i} + \frac{4}{\sqrt{14}}\hat{j} + \frac{12}{\sqrt{14}}\hat{k}$$

$$\vec{u} + \vec{w} = \left(2\sqrt{3} - \frac{8}{\sqrt{14}}\right)\hat{i} + \left(2 + \frac{4}{\sqrt{14}}\right)\hat{j} + \frac{12}{\sqrt{14}}\hat{k}$$

3. (2 points) Find a unit vector orthogonal to $8\hat{i} - 7\hat{j} + 2\hat{k}$.

$$\approx 1.326\hat{i} + 3.069\hat{j} + 3.207\hat{k}$$

$\hat{i} - 4\hat{k}$ IS ORTHOGONAL TO IT

SINCE THE DOT PRODUCT IS 0.

$$\frac{1}{\sqrt{17}}(\hat{i} - 4\hat{k})$$

4. (2 points) Suppose \vec{u} and \vec{v} are orthogonal to \vec{w} . Show, or explain how you know, that $3\vec{u} - 2\vec{v}$ is orthogonal to \vec{w} .

$$\vec{u} \cdot \vec{w} = 0 \text{ AND } \vec{v} \cdot \vec{w} = 0 \text{ SINCE } \vec{u} \text{ & } \vec{v} \text{ ARE ORTHOGONAL TO } \vec{w}.$$

$$\vec{w} \cdot (3\vec{u} - 2\vec{v}) = 3(\vec{w} \cdot \vec{u}) - 2(\vec{w} \cdot \vec{v})$$

$$= 3(0) - 2(0) = 0$$

$\Rightarrow \vec{w}$ IS ORTHOGONAL TO $3\vec{u} - 2\vec{v}$.