Math 173 - Quiz 2

February 6, 2014

Name key

Score _____

Show all work to receive full credit. Supply explanations when necessary.

1. (8 points) Let $\vec{u} = \hat{\imath} - 3\hat{\jmath} + 2\hat{k}$ and $\vec{w} = 5\hat{\imath} + 2\hat{\jmath} - \hat{k}$.

   (a) Find the projection of $\vec{w}$ onto $\vec{u}$.

$$\text{proj}_{\vec{u}}\,\vec{w} = \frac{\vec{u}\cdot\vec{w}}{\vec{u}\cdot\vec{u}}\,\vec{u} = \frac{5-6-2}{1+9+4}\,\vec{u} = -\frac{3}{14}\left(\hat{\imath}-3\hat{\jmath}+2\hat{k}\right)$$

$$= \boxed{-\frac{3}{14}\hat{\imath} + \frac{9}{14}\hat{\jmath} - \frac{3}{7}\hat{k}}$$

$\vec{u}\cdot\vec{w} = 25+4+1 = 30$

   (b) Find the angle between $\vec{u}$ and $\vec{w}$.

$$\vec{u}\cdot\vec{w} = \|\vec{u}\|\,\|\vec{w}\|\cos\theta$$

$$\cos\theta = \frac{\vec{u}\cdot\vec{w}}{\|\vec{u}\|\,\|\vec{w}\|} = \frac{-3}{\sqrt{14}\sqrt{30}} \qquad \theta = \cos^{-1}\left(\frac{-3}{\sqrt{14}\sqrt{30}}\right) \approx \boxed{98.4^\circ}$$

   (c) Find a vector orthogonal to both $\vec{u}$ and $\vec{w}$.

$$\vec{u}\times\vec{w} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 1 & -3 & 2 \\ 5 & 2 & -1 \end{vmatrix} = \hat{\imath}(-1) - \hat{\jmath}(-11) + \hat{k}(17)$$

$$= \boxed{-\hat{\imath} + 11\hat{\jmath} + 17\hat{k}}$$

   (d) Without doing any computations, explain why it is impossible to find a unit vector in the direction of $\vec{u}\times\vec{u}$.

$\vec{u}\times\vec{u} = \vec{0}$ IT IS NOT POSSIBLE TO NORMALIZE THE ZERO VECTOR.

2. (2 points) If $\vec{u}\cdot\vec{v} = \vec{u}\cdot\vec{w}$, must it be true that $\vec{v} = \vec{w}$?

No, $\vec{u}\cdot\vec{v} = \vec{u}\cdot\vec{w} \Rightarrow \vec{u}\cdot(\vec{v}-\vec{w}) = 0$

$\vec{u}$ AND $\vec{v}-\vec{w}$ BEING ORTHOGONAL DOES NOT MAKE $\vec{v} = \vec{w}$.

Ex LET $\vec{u} = \hat{\imath}+\hat{\jmath}+\hat{k}$

$\vec{v} = 5\hat{\imath}-2\hat{\jmath}-3\hat{k}$

$\vec{w} = 2\hat{\imath}+3\hat{\jmath}-5\hat{k}$

$\vec{u}\cdot\vec{v} = \vec{u}\cdot\vec{w} = 0$ BUT $\vec{v} \neq \vec{w}$