Math 173 - Quiz 5

March 6, 2014

Name key

Score _____

Show all work to receive full credit. Supply explanations when necessary.

1. (3 points) In Moscow in 1987, Natalya Lisouskaya set a women's world record by putting an 8 lb 13 oz shot 73 ft 10 in. Assuming that she launched the shot at a 40° angle 6.5 ft above the ground, what was the shot's initial speed? (Use $g = 32\,\text{ft/s}^2$.)

$$\vec{r}(t) = v_0 \cos 40° t\,\hat{i} + \left(-16t^2 + v_0 \sin 40° t + 6.5\right)\hat{j}$$

$$\approx 0.766 v_0 t\,\hat{i} + \left(-16t^2 + 0.643 v_0 t + 6.5\right)\hat{j}$$

$$0.766 v_0 t = 73\tfrac{10}{12} \Rightarrow v_0 t \approx 96.383$$

$$-16t^2 + 0.643 v_0 t + 6.5 = 0$$

$$-16t^2 = -68.454 \Rightarrow t \approx 2.07$$

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$$v_0 \approx 46.6 \text{ FT/S}$$

2. (4 points) Let $\vec{r}(t) = (6\sin 2t)\hat{i} + (6\cos 2t)\hat{j} + 5t\hat{k}$. Find $\hat{T}$ and $\hat{N}$.

$$\vec{r}\,'(t) = 12\cos 2t\,\hat{i} - 12\sin 2t\,\hat{j} + 5\hat{k}$$

$$\|\vec{r}\,'(t)\| = \sqrt{144 + 25} = 13$$

$$\hat{T}(t) = \frac{1}{13}\left(12\cos 2t\,\hat{i} - 12\sin 2t\,\hat{j} + 5\hat{k}\right)$$

$$\hat{T}'(t) = \frac{1}{13}\left(-24\sin 2t\,\hat{i} - 24\cos 2t\,\hat{j}\right)$$

$$\|\hat{T}'(t)\| = \frac{24}{13}$$

$$\hat{N}(t) = \frac{\hat{T}'(t)}{\|\hat{T}'(t)\|}$$

$$\hat{N}(t) = -\sin 2t\,\hat{i} - \cos 2t\,\hat{j}$$

3. (3 points) Let $\vec{r}(t) = 6t^3\hat{i} + (t - 2t^2)\hat{j} + e^{t/2}\hat{k}$. Set up the integral required to find the length of the graph of $\vec{r}$ over the interval from $t = 0$ to $t = 1$. Use your calculator to approximate the value of your integral.

$$\text{ARC LENGTH} = \int_0^1 \sqrt{(18t^2)^2 + (1-4t)^2 + \left(\tfrac{1}{2}e^{t/2}\right)^2}\,dt$$

$$\approx 6.278$$