

# Math 173 - Quiz 7

April 10, 2014

Name key \_\_\_\_\_  
Score \_\_\_\_\_

Show all work to receive full credit. Supply explanations when necessary.

1. (2 points) Find an equation of the plane tangent to the graph of  $f(x, y) = \sqrt{xy^2 + 6y^2}$  at the point  $(3, -1, 3)$ .

$$\text{LET } F(x, y, z) = z - \sqrt{xy^2 + 6y^2}$$

$$\vec{N} = -\frac{1}{6}\hat{i} + 3\hat{j} + \hat{k}$$

$\vec{\nabla} F(x, y, z)$  IS NORMAL TO  $F(x, y, z) = 0$

POINT  $(3, -1, 3)$

AT  $(3, -1, 3)$ .

$$\vec{\nabla} F(x, y, z) = -\frac{1}{2}(xy^2 + 6y^2)^{-1/2}(y^2)\hat{i}$$

$$-\frac{1}{2}(xy^2 + 6y^2)^{-1/2}(2xy + 12y)\hat{j} + \hat{k}$$

$$\vec{\nabla} F(3, -1, 3) = -\frac{1}{6}\hat{i} + 3\hat{j} + \hat{k}$$

PLANE IS  

$$-\frac{1}{6}(x-3) + 3(y+1) + (z-3) = 0$$

2. (2 points) Suppose the temperature at the point  $(x, y)$  on a thin sheet of metal is given by

$$T(x, y) = \frac{100(1 + 3x + 2y)}{1 + 2x^2 + 3y^2},$$

where  $T$  is measured in degrees Fahrenheit. In what direction is the temperature increasing most rapidly at the point  $(1, 2)$ ?

$$\begin{aligned} \vec{\nabla} T(x, y) &= \frac{(1+2x^2+3y^2)(300)-100(1+3x+2y)(4x)}{(1+2x^2+3y^2)^2}\hat{i} \\ &\quad + \frac{(1+2x^2+3y^2)(200)-100(1+3x+2y)(6y)}{(1+2x^2+3y^2)^2}\hat{j} \end{aligned}$$

$$\begin{aligned} \vec{\nabla} T(1, 2) &= \frac{(15)(300)-100(8)(4)}{15^2}\hat{i} + \frac{(15)(200)-100(8)(12)}{15^2}\hat{j} \\ &= \frac{52}{9}\hat{i} - \frac{88}{3}\hat{j} = 5.\overline{7}\hat{i} - 29.\overline{3}\hat{j} \end{aligned}$$

Normalized  $\vec{\nabla} T(1, 2) \approx 0.19326\hat{i} - 0.98115\hat{j}$

3. (3 points) Find the linearization of  $f(x, y) = xe^{-y}$  at the point  $(2, 0)$ . Then use it to approximate  $f(1.98, 0.1)$ .

$$f_x(x, y) = e^{-y}$$

LINERIZATION IS

$$f_y(x, y) = -xe^{-y}$$

$$L(x, y) = f(2, 0) + f_x(2, 0)(x-2) + f_y(2, 0)(y-0)$$

$$f_x(2, 0) = 1$$

$$L(x, y) = 2 + 1(x-2) - 2(y-0)$$

$$f_y(2, 0) = -2$$

$$\boxed{L(x, y) = x - 2y}$$

$$f(1.98, 0.1) \approx L(1.98, 0.1) = 1.98 - 0.2$$

$$= \boxed{1.78}$$

4. (3 points) Find and classify the critical points of  $f(x, y) = x^3 - 3xy + y^3 + 3$ .

$$f_x(x, y) = 3x^2 - 3y = 0 \Rightarrow y = x^2$$

$$f_y(x, y) = -3x + 3y^2 = 0 \Rightarrow x = y^2 \Rightarrow x = x^4 \Rightarrow x = 0 \text{ or } x = 1$$

$$\downarrow \quad \downarrow$$

$$y = 0 \text{ or } y = 1$$

THERE ARE TWO

CRITICAL PTS:

$$D(x, y) = \begin{vmatrix} 6x & -3 \\ -3 & 6y \end{vmatrix} = 36xy - 9$$

$(0, 0)$  AND  $(1, 1)$ .

$$D(0, 0) = -9 \Rightarrow (0, 0) \text{ CORRESPONDS}$$

TO A SADDLE PT

$$D(1, 1) = 27 \text{ AND } f_{xx}(1, 1) = 6$$

$\Rightarrow (1, 1)$  CORRESPONDS TO  
A RELATIVE MIN.

$(0, 0, 3)$  IS A SADDLE PT.

$f(1, 1) = 0$  IS A REL. MIN.