Math 173 - Final Exam

May 12, 2014

Name ______________________

Score ______________

Show all work. Supply explanations when necessary. Unless otherwise indicated, each problem is worth 12 points.

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1. Sketch the region of integration, reverse the order of integration, and **evaluate the iterated integral by hand**.

$$\int_0^4 \int_{\sqrt{x}}^2 \frac{x}{y^5+1}\, dy\, dx$$

2. Find and classify all relative extreme values and saddle points.

$$f(x, y) = x^2 + xy + 2y^2 + x - 3y + 10$$

3. A plane passes through the points $P\,(2,1,1)$, $Q\,(0,4,1)$ and $R\,(-2,1,4)$. Find a set of parametric equations for the line normal to the plane and passing through $(5,-9,3)$.

4. The curves described by the vector-valued functions $\vec{r_1}(t) = t\hat{\imath} + (t^2+3)\hat{\jmath} + (6-5t)\hat{k}$ and $\vec{r_2}(t) = t^2\hat{\imath} + (2t+2)\hat{\jmath} + t^3\hat{k}$ intersect at the point $(1,4,1)$. Find the angle of intersection of the curves. (Hint: Find the angle between the velocity vectors at the point of intersection.)

5. (14 points) Evaluate the limit or show that it does not exist.

(a) $\displaystyle\lim_{(x,y)\to(1,1)} \frac{5x-5y}{y^2-x^2}$

(b) $\displaystyle\lim_{(x,y)\to(0,0)} \frac{x^2y}{x^4+y^2}$

6. Suppose w depends on x, y, and z according to $w = x\cos(yz)$ and x, y, and z depend on s and t:

$$x = s^2, \qquad y = t^2, \qquad z = s - 2t.$$

Write the chain rule formula for $\partial w/\partial t$. Then use your formula to find $\partial w/\partial t$ at $(s,t) = (\pi, 0)$.

7. Let S be the space region bounded by the paraboloids $z = 2x^2 + 2y^2$ and $z = 6 - x^2 - y^2$. Evaluate the following triple integral **by hand**. (Hint: It's best to use cylindrical coordinates.)

$$\iiint_S (x^2 + y^2)\, dV$$

8. The quarterback of a football team throws the ball with an initial speed of 54 feet per second, at an angle of 36°, and at a height of 6 feet above the playing field. How far downfield has the ball traveled at the moment when it reaches its maximum height? (Ignore air resistance and use $g = 32\,\text{ft/sec}^2$.)

9. The temperature at the point (x, y, z) is given by

$$T(x, y, z) = xz^2 \cos(\pi y)$$

where T is measured in °C and x, y, and z in centimeters.

(a) At the point $(2, -1, 2)$, in what direction does the temperature decrease the fastest?

(b) Find the rate of change of temperature at the point $(2, -1, 2)$ in the direction toward the point $(3, -3, 3)$.

10. Let $\vec{r}(t) = t^2\hat{\imath} + (\sin t - t\cos t)\hat{\jmath} + (\cos t + t\sin t)\hat{k}$. Find the unit tangent vector and the principal unit normal vector. (Hint: If you're doing everything correctly, the computations should not be messy.)

11. (13 points) A thin plate is bounded by the graphs of $y = x^2$ and $y = 2 - x$. The density of the plate at the point (x, y) is given by $\rho(x, y) = 5 + xy$. Sketch the thin plate. Then set up the three iterated integrals required to find the center of mass of the plate. Do not evaluate the integrals.

12. (15 points) Let P and Q be the points $(3, 1, 5)$ and $(-2, 2, 7)$, respectively.

(a) Find a vector of length 4 in the direction of $\overrightarrow{PQ}$.

(b) Find a unit vector in the xy-plane that is orthogonal to $\overrightarrow{PQ}$.

(c) Find the projection of $\overrightarrow{PQ}$ onto $\hat{\imath} + \hat{\jmath} - \hat{k}$.